7 Right Triangles and Trigonometry

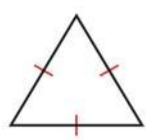
Before

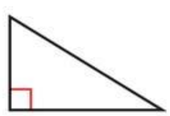
In previous courses and in Chapters 1–6, you learned the following skills, which you'll use in Chapter 7: classifying triangles, simplifying radicals, and solving proportions.

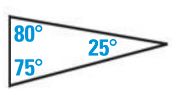
Prerequisite Skills

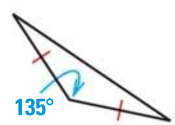
VOCABULARY CHECK

Name the triangle shown.

1. 

2. 

3. 

4. 

SKILLS AND ALGEBRA CHECK

Simplify the radical. *(Review p. 874 for 7.1, 7.2, 7.4.)*

5. $\sqrt{45}$ **6.** $(3\sqrt{7})^2$ **7.** $\sqrt{3} \cdot \sqrt{5}$ **8.** $\frac{7}{\sqrt{2}}$

Solve the proportion. *(Review p. 356 for 7.3, 7.5–7.7.)*

9. $\frac{3}{x} = \frac{12}{16}$ **10.** $\frac{2}{3} = \frac{x}{18}$ **11.** $\frac{x+5}{4} = \frac{1}{2}$ **12.** $\frac{x+4}{x-4} = \frac{6}{5}$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 7, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 493. You will also use the key vocabulary listed below.

Big Ideas

1. **Using the Pythagorean Theorem and its converse**
2. **Using special relationships in right triangles**
3. **Using trigonometric ratios to solve right triangles**

KEY VOCABULARY

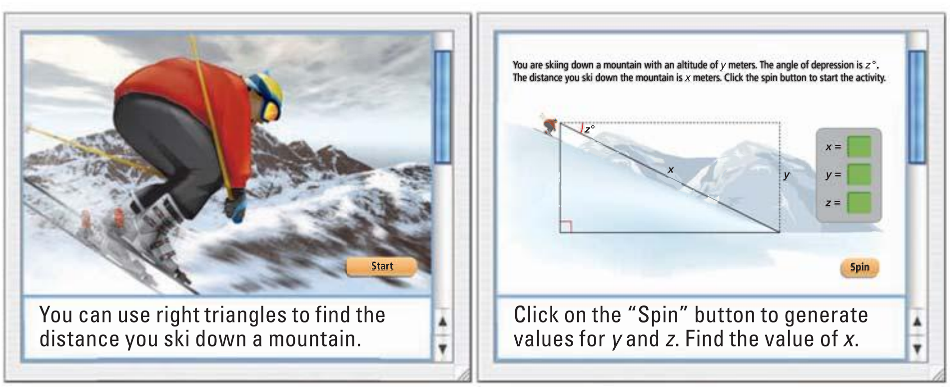
- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

Why?

You can use trigonometric ratios to find unknown side lengths and angle measures in right triangles. For example, you can find the length of a ski slope.

Animated Geometry

The animation illustrated below for Example 4 on page 475 helps you answer this question: How far will you ski down the mountain?



Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 7: pages 434, 442, 450, 460, and 462

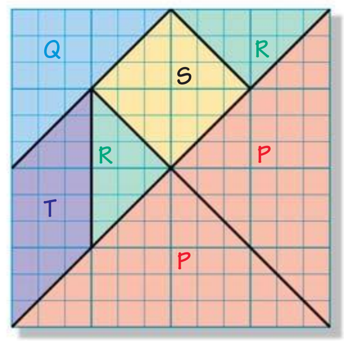
7.1 Pythagorean Theorem

MATERIALS • graph paper • ruler • pencil • scissors

QUESTION What relationship exists among the sides of a right triangle?

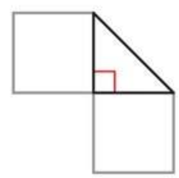
Recall that a square is a four sided figure with four right angles and four congruent sides.

EXPLORE Make and use a tangram set

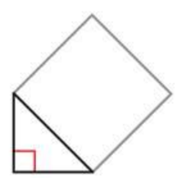


STEP 1 ***Make a tangram set*** On your graph paper, copy the tangram set as shown. Label each piece with the given letters. Cut along the solid black lines to make seven pieces.

STEP 2 ***Trace a triangle*** On another piece of paper, trace one of the large triangles P of the tangram set.



STEP 3 ***Assemble pieces along the legs*** Use all of the tangram pieces to form two squares along the legs of your triangle so that the length of each leg is equal to the side length of the square. Trace all of the pieces.



STEP 4 ***Assemble pieces along the hypotenuse*** Use all of the tangram pieces to form a square along the hypotenuse so that the side length of the square is equal to the length of the hypotenuse. Trace all of the pieces.

DRAW CONCLUSIONS Use your observations to complete these exercises

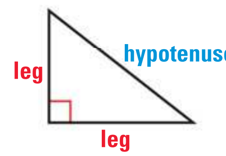
1. Find the sum of the areas of the two squares formed in Step 3. Let the letters labeling the figures represent the area of the figure. How are the side lengths of the squares related to Triangle P?
2. Find the area of the square formed in Step 4. How is the side length of the square related to Triangle P?
3. Compare your answers from Exercises 1 and 2. Make a conjecture about the relationship between the legs and hypotenuse of a right triangle.
4. The triangle you traced in Step 2 is an isosceles right triangle. Why? Do you think that your conjecture is true for all isosceles triangles? Do you think that your conjecture is true for all right triangles? *Justify* your answers.

7.1 Apply the Pythagorean Theorem

Before	You learned about the relationships within triangles.
Now	You will find side lengths in right triangles.
Why?	So you can find the shortest distance to a campfire, as in Ex. 35.

Key Vocabulary
- **Pythagorean triple**
- **right triangle,** *p. 217*
- **leg of a right triangle,** *p. 241*
- **hypotenuse,** *p. 241*

One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras (around 500 B.C.). This theorem can be used to find information about the lengths of the sides of a right triangle.

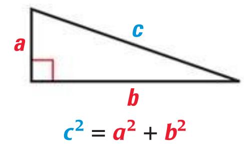


THEOREM *For Your Notebook*

THEOREM 7.1 Pythagorean Theorem

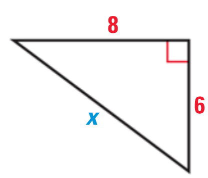
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Proof: p. 434; Ex. 32, p. 455



EXAMPLE 1 Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.



Solution

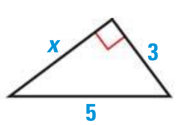
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$x^2 = 6^2 + 8^2$	Substitute.
$x^2 = 36 + 64$	Multiply.
$x^2 = 100$	Add.
$x = 10$	Find the positive square root.

ABBREVIATE
In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

GUIDED PRACTICE for Example 1

Identify the unknown side as a *leg* or *hypotenuse*. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.

1.



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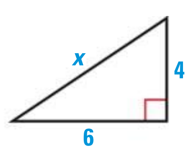
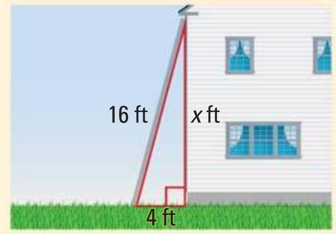


EXAMPLE 2 Standardized Test Practice

A 16 foot ladder rests against the side of the house, and the base of the ladder is 4 feet away. Approximately how high above the ground is the top of the ladder?

Ⓐ 240 feet Ⓑ 20 feet

Ⓒ 16.5 feet Ⓓ 15.5 feet



Solution

$\left(\text{Length of ladder}\right)^2 = \left(\text{Distance from house}\right)^2 + \left(\text{Height of ladder}\right)^2$

$16^2 = 4^2 + x^2$ **Substitute.**

$256 = 16 + x^2$ **Multiply.**

$240 = x^2$ **Subtract 16 from each side.**

$\sqrt{240} = x$ **Find positive square root.**

$15.491 \approx x$ **Approximate with a calculator.**

The ladder is resting against the house at about 15.5 feet above the ground.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

APPROXIMATE

In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

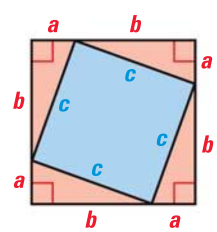
✓ GUIDED PRACTICE for Example 2

3. The top of a ladder rests against a wall, 23 feet above the ground. The base of the ladder is 6 feet away from the wall. What is the length of the ladder?

4. The Pythagorean Theorem is only true for what type of triangle?

PROVING THE PYTHAGOREAN THEOREM There are many proofs of the Pythagorean Theorem. An informal proof is shown below. You will write another proof in Exercise 32 on page 455.

In the figure at the right, the four right triangles are congruent, and they form a small square in the middle. The area of the large square is equal to the area of the four triangles plus the area of the smaller square.



$\left(\text{Area of large square}\right) = \left(\text{Area of four triangles}\right) + \left(\text{Area of smaller square}\right)$

$(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$ **Use area formulas.**

$a^2 + 2ab + b^2 = 2ab + c^2$ **Multiply.**

$a^2 + b^2 = c^2$ **Subtract 2*ab* from each side.**

REVIEW AREA

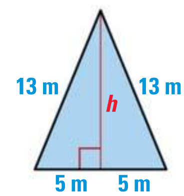
Recall that the area of a square with side length s is $A = s^2$. The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.

Animated Geometry at classzone.com

EXAMPLE 3 Find the area of an isosceles triangle

Find the area of the isosceles triangle with side lengths 10 meters, 13 meters, and 13 meters.

Solution



STEP 1 **Draw** a sketch. By definition, the length of an altitude is the height of a triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.

STEP 2 **Use** the Pythagorean Theorem to find the height of the triangle.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$13^2 = 5^2 + h^2$	Substitute.
$169 = 25 + h^2$	Multiply.
$144 = h^2$	Subtract 25 from each side.
$12 = h$	Find the positive square root.

STEP 3 **Find** the area.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(10)(12) = 60 \text{ m}^2$$

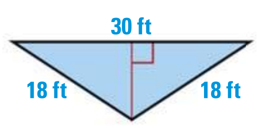
▶ The area of the triangle is 60 square meters.

READ TABLES
You may find it helpful to use the Table of Squares and Square Roots on p. 924.

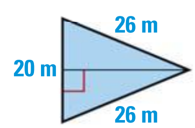
✓ GUIDED PRACTICE for Example 3

Find the area of the triangle.

5.



6.



PYTHAGOREAN TRIPLES A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

STANDARDIZED TESTS
You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

KEY CONCEPT *For Your Notebook*

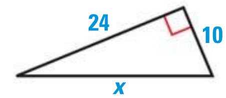
Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

EXAMPLE 4 Find the length of a hypotenuse using two methods

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 5, 12, 13. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: $5 \cdot 2 = 10$ and $12 \cdot 2 = 24$. So, the length of the hypotenuse is $13 \cdot 2 = 26$.

Method 2: Use the Pythagorean Theorem.

$x^2 = 10^2 + 24^2$ **Pythagorean Theorem**

$x^2 = 100 + 576$ **Multiply.**

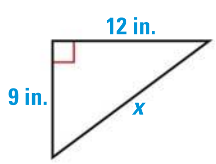
$x^2 = 676$ **Add.**

$x = 26$ **Find the positive square root.**

✓ GUIDED PRACTICE for Example 4

Find the unknown side length of the right triangle using the Pythagorean Theorem. Then use a Pythagorean triple.

7.



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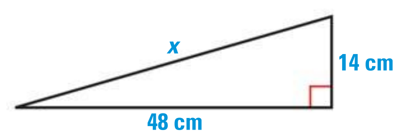


7.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 27, 33, and 36

◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

SKILL PRACTICE

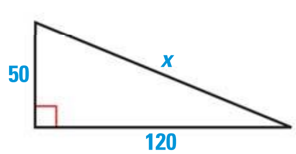
1. **VOCABULARY** Copy and complete: A set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$ is called a _?_.

2. ★ **WRITING** *Describe* the information you need to have in order to use the Pythagorean Theorem to find the length of a side of a triangle.

EXAMPLE 1 on p. 433 for Exs. 3–7

ALGEBRA **Find the length of the hypotenuse of the right triangle.**

3.



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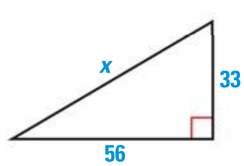

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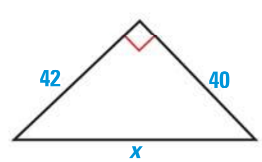
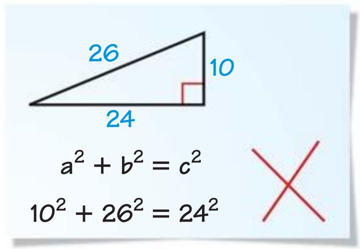


ERROR ANALYSIS ***Describe*** **and correct the error in using the Pythagorean Theorem.**

6.



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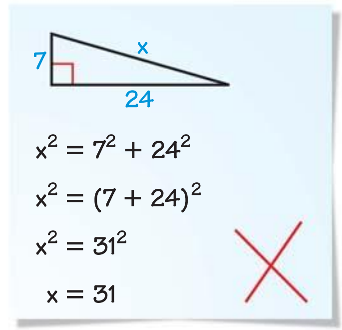


EXAMPLE 2
on p. 434
for Exs. 8–10

FINDING A LENGTH **Find the unknown leg length *x*.**

8.

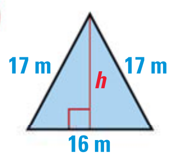
9.

10.

EXAMPLE 3
on p. 435
for Exs. 11–13

FINDING THE AREA **Find the area of the isosceles triangle.**

11.



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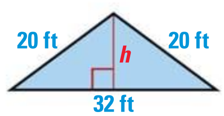

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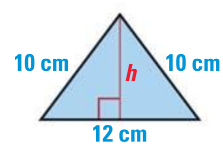
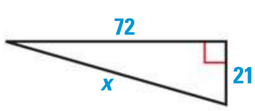


EXAMPLE 4
on p. 436
for Exs. 14–17

FINDING SIDE LENGTHS **Find the unknown side length of the right triangle using the Pythagorean Theorem or a Pythagorean triple.**

14.



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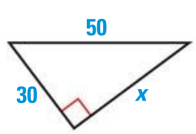

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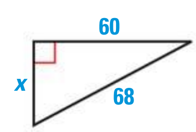


17. ★ **MULTIPLE CHOICE** What is the length of the hypotenuse of a right triangle with leg lengths of 8 inches and 15 inches?

Ⓐ 13 inches Ⓑ 17 inches Ⓒ 21 inches Ⓓ 25 inches

PYTHAGOREAN TRIPLES **The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.**

18. 24 and 51

19. 20 and 25

20. 28 and 96

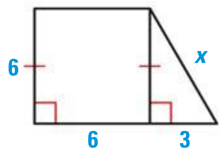
21. 20 and 48

22. 75 and 85

23. 72 and 75

FINDING SIDE LENGTHS **Find the unknown side length x. Write your answer in simplest radical form.**

24.



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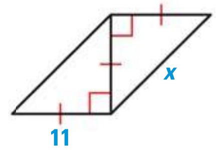

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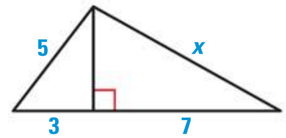


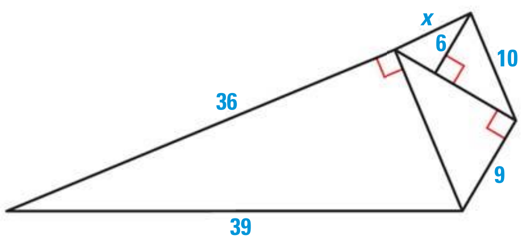
27. ★ **MULTIPLE CHOICE** What is the area of a right triangle with a leg length of 15 feet and a hypotenuse length of 39 feet?

Ⓐ 270 ft^2 Ⓑ 292.5 ft^2 Ⓒ 540 ft^2 Ⓓ 585 ft^2

28. **ALGEBRA** Solve for x if the lengths of the two legs of a right triangle are $2x$ and $2x + 4$, and the length of the hypotenuse is $4x - 4$.

CHALLENGE **In Exercises 29 and 30, solve for x.**

29.



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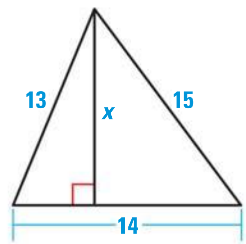

PROBLEM SOLVING

EXAMPLE 2
on p. 434
for Exs. 31–32

31. **BASEBALL DIAMOND** In baseball, the distance of the paths between each pair of consecutive bases is 90 feet and the paths form right angles. How far does the ball need to travel if it is thrown from home plate directly to second base?

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32. **APPLE BALLOON** You tie an apple balloon to a stake in the ground. The rope is 10 feet long. As the wind picks up, you observe that the balloon is now 6 feet away from the stake. How far above the ground is the balloon now?

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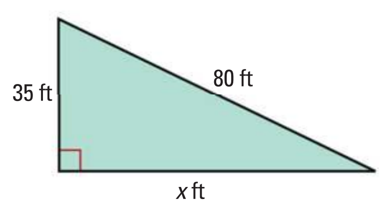
33. ★ **SHORT RESPONSE** Three side lengths of a right triangle are 25, 65, and 60. *Explain* how you know which side is the hypotenuse.

34. **MULTI-STEP PROBLEM** In your town, there is a field that is in the shape of a right triangle with the dimensions shown.

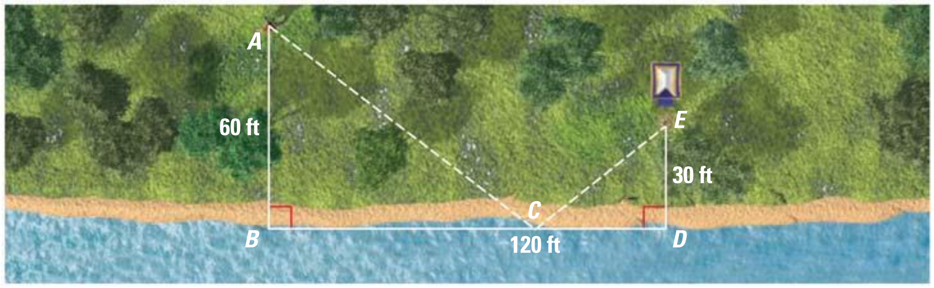
a. Find the perimeter of the field.

b. You are going to plant dogwood seedlings about every ten feet around the field's edge. How many trees do you need?

c. If each dogwood seedling sells for \$12, how much will the trees cost?



35. **MULTIPLE REPRESENTATIONS** As you are gathering leaves for a science project, you look back at your campsite and see that the campfire is not completely out. You want to get water from a nearby river to put out the flames with the bucket you are using to collect leaves. Use the diagram and the steps below to determine the shortest distance you must travel.



a. Making a Table Make a table with columns labeled *BC*, *AC*, *CE*, and *AC* + *CE*. Enter values of *BC* from 10 to 120 in increments of 10.

b. Calculating Values Calculate *AC*, *CE*, and *AC* + *CE* for each value of *BC*, and record the results in the table. Then, use your table of values to determine the shortest distance you must travel.

c. Drawing a Picture Draw an accurate picture to scale of the shortest distance.

36. ★ **SHORT RESPONSE** *Justify* the Distance Formula using the Pythagorean Theorem.

37. **PROVING THEOREM 4.5** Find the Hypotenuse-Leg (HL) Congruence Theorem on page 241. Assign variables for the side lengths in the diagram. Use your variables to write GIVEN and PROVE statements. Use the Pythagorean Theorem and congruent triangles to prove Theorem 4.5.

38. **CHALLENGE** Trees grown for sale at nurseries should stand at least five feet from one another while growing. If the trees are grown in parallel rows, what is the smallest allowable distance between rows?

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.2 in Exs. 39–42.

Evaluate the expression. *(p. 874)*

39. $(\sqrt{7})^2$ **40.** $(4\sqrt{3})^2$ **41.** $(-6\sqrt{81})^2$ **42.** $(-8\sqrt{2})^2$

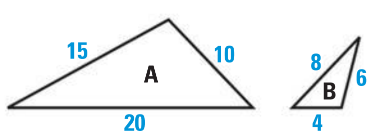
Describe **the possible lengths of the third side of the triangle given the lengths of the other two sides.** *(p. 328)*

43. 3 feet, 6 feet **44.** 5 inches, 11 inches **45.** 14 meters, 21 meters

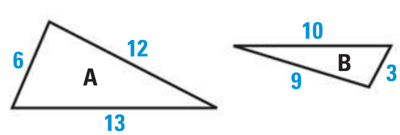
46. 12 inches, 27 inches **47.** 18 yards, 18 yards **48.** 27 meters, 39 meters

Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A. *(p. 388)*

49.



50.



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Keystrokes

7.2 Converse of the Pythagorean Theorem

MATERIALS • graphing calculator or computer

QUESTION **How can you use the side lengths in a triangle to classify the triangle by its angle measures?**

You can use geometry drawing software to construct and measure triangles.

EXPLORE Construct a triangle

STEP 1 ***Draw a triangle*** Draw any $\triangle ABC$ with the largest angle at C. Measure $\angle C$, $\overline{AB}$, $\overline{AC}$, and $\overline{CB}$.

STEP 2 ***Calculate*** Use your measurements to calculate AB^2, AC^2, CB^2, and $(AC^2 + CB^2)$.

STEP 3 ***Complete a table*** Copy the table below and record your results in the first row. Then move point A to different locations and record the values for each triangle in your table. Make sure $\overline{AB}$ is always the longest side of the triangle. Include triangles that are acute, right, and obtuse.

$m\angle C$	AB	AB^2	AC	CB	$AC^2 + CB^2$
76°	5.2	27.04	4.5	3.8	34.69
?	?	?	?	?	?
?	?	?	?	?	?

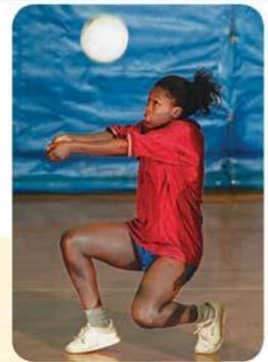
DRAW CONCLUSIONS Use your observations to complete these exercises

1. The Pythagorean Theorem states that "In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs." Write the Pythagorean Theorem in if-then form. Then write its converse.
2. Is the converse of the Pythagorean Theorem true? *Explain.*
3. Make a conjecture about the relationship between the measure of the largest angle in a triangle and the squares of the side lengths.

Copy and complete the statement.

4. If $AB^2 > AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.
5. If $AB^2 < AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.
6. If $AB^2 = AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.

7.2 Use the Converse of the Pythagorean Theorem



Before	You used the Pythagorean Theorem to find missing side lengths.
Now	You will use its converse to determine if a triangle is a right triangle.
Why?	So you can determine if a volleyball net is set up correctly, as in Ex. 38.

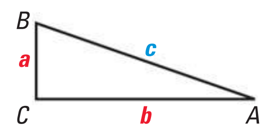
Key Vocabulary
- **acute triangle,** *p. 217*
- **obtuse triangle,** *p. 217*

The converse of the Pythagorean Theorem is also true. You can use it to verify that a triangle with given side lengths is a right triangle.

THEOREM *For Your Notebook*

THEOREM 7.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



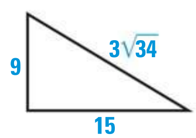
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Proof: Ex. 42, p. 446

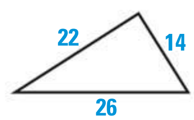
EXAMPLE 1 Verify right triangles

Tell whether the given triangle is a right triangle.

a.



b.



Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(3\sqrt{34})^2 \stackrel{?}{=} 9^2 + 15^2$

$9 \cdot 34 \stackrel{?}{=} 81 + 225$

$306 = 306$ ✓

The triangle is a right triangle.

b. $26^2 \stackrel{?}{=} 22^2 + 14^2$

$676 \stackrel{?}{=} 484 + 196$

$676 \neq 680$

The triangle is not a right triangle.

REVIEW ALGEBRA
Use a square root table or a calculator to find the decimal representation. So, $3\sqrt{34} \approx 17.493$ is the length of the longest side in part (a).

✓ GUIDED PRACTICE for Example 1

Tell whether a triangle with the given side lengths is a right triangle.

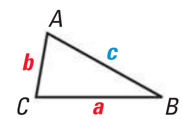
1. $4, 4\sqrt{3}, 8$ **2.** 10, 11, and 14 **3.** 5, 6, and $\sqrt{61}$

CLASSIFYING TRIANGLES The Converse of the Pythagorean Theorem is used to verify that a given triangle is a right triangle. The theorems below are used to verify that a given triangle is acute or obtuse.

THEOREMS *For Your Notebook*

THEOREM 7.3

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle ABC is an acute triangle.

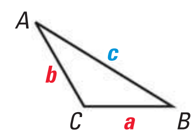


If $c^2 < a^2 + b^2$, then the triangle ABC is acute.

Proof: Ex. 40, p. 446

THEOREM 7.4

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle ABC is an obtuse triangle.



If $c^2 > a^2 + b^2$, then triangle ABC is obtuse.

Proof: Ex. 41, p. 446

EXAMPLE 2 Classify triangles

Can segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle? If so, would the triangle be *acute, right,* or *obtuse*?

APPLY THEOREMS
The Triangle Inequality Theorem on page 330 states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Solution

STEP 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

$4.3 + 5.2 = 9.5$ $\quad$ $4.3 + 6.1 = 10.4$ $\quad$ $5.2 + 6.1 = 11.3$

$9.5 > 6.1$ $\quad$ $10.4 > 5.2$ $\quad$ $11.3 > 4.3$

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet can form a triangle.

STEP 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \overset{?}{\_\_} a^2 + b^2$		**Compare c^2 with $a^2 + b^2$.**
$6.1^2 \overset{?}{\_\_} 4.3^2 + 5.2^2$		**Substitute.**
$37.21 \overset{?}{\_\_} 18.49 + 27.04$		**Simplify.**
$37.21 < 45.53$		**c^2 is less than $a^2 + b^2$.**

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

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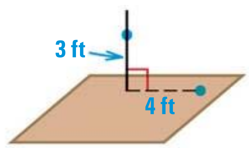
EXAMPLE 3 Use the Converse of the Pythagorean Theorem

CATAMARAN You are part of a crew that is installing the mast on a catamaran. When the mast is fastened properly, it is perpendicular to the trampoline deck. How can you check that the mast is perpendicular using a tape measure?

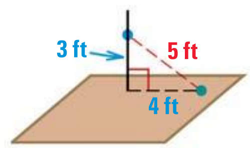
Solution

To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

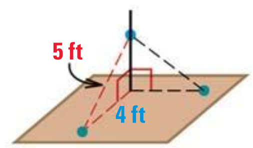
Think of the mast as a line and the deck as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the mast is perpendicular to different lines on the deck.



First place a mark 3 feet up the mast and a mark on the deck 4 feet from the mast.



Use the tape measure to check that the distance between the two marks is 5 feet. The mast makes a right angle with the line on the deck.



Finally, repeat the procedure to show that the mast is perpendicular to another line on the deck.

GUIDED PRACTICE for Example 2 and 3

4. Show that segments with lengths 3, 4, and 6 can form a triangle and classify the triangle as *acute, right,* or *obtuse*.

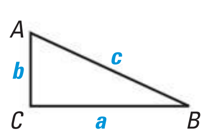
5. **WHAT IF?** In Example 3, could you use triangles with side lengths 2, 3, and 4 to verify that you have perpendicular lines? *Explain*.

CLASSIFYING TRIANGLES You can use the theorems from this lesson to classify a triangle as acute, right, or obtuse based on its side lengths.

CONCEPT SUMMARY *For Your Notebook*

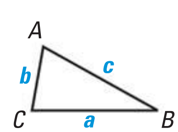
Methods for Classifying a Triangle by Angles Using its Side Lengths

Theorem 7.2



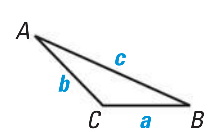
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

Theorem 7.4



If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

7.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 25, 32, 38, 39, and 43

SKILL PRACTICE

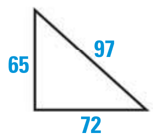
1. VOCABULARY What is the longest side of a right triangle called?

2. ★ WRITING *Explain* how the side lengths of a triangle can be used to classify it as acute, right, or obtuse.

EXAMPLE 1
on p. 441
for Exs. 3–14

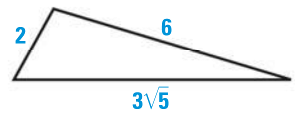
VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.**

3.

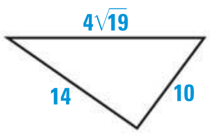


4.

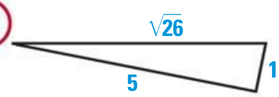
5.



6.



7.



8.

VERIFYING RIGHT TRIANGLES **Tell whether the given side lengths of a triangle can represent a right triangle.**

9. 9, 12, and 15

10. 9, 10, and 15

11. 36, 48, and 60

12. 6, 10, and $2\sqrt{34}$

13. 7, 14, and $7\sqrt{5}$

14. 10, 12, and 20

EXAMPLE 2
on p. 442
for Exs. 15–23

CLASSIFYING TRIANGLES **In Exercises 15–23, decide if the segment lengths form a triangle. If so, would the triangle be *acute, right,* or *obtuse?***

15. 10, 11, and 14

16. 10, 15, and $5\sqrt{13}$

17. 24, 30, and $6\sqrt{43}$

18. 5, 6, and 7

19. 12, 16, and 20

20. 8, 10, and 12

21. 15, 20, and 36

22. 6, 8, and 10

23. 8.2, 4.1, and 12.2

24. ★ MULTIPLE CHOICE Which side lengths do not form a right triangle?

Ⓐ 5, 12, 13 Ⓑ 10, 24, 28 Ⓒ 15, 36, 39 Ⓓ 50, 120, 130

25. ★ MULTIPLE CHOICE What type of triangle has side lengths of 4, 7, and 9?

Ⓐ Acute scalene Ⓑ Right scalene

Ⓒ Obtuse scalene Ⓓ None of the above

26. ERROR ANALYSIS A student tells you that if you double all the sides of a right triangle, the new triangle is obtuse. *Explain* why this statement is incorrect.

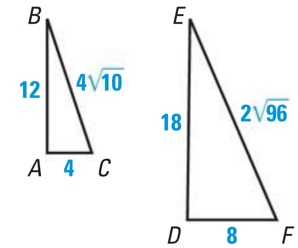
GRAPHING TRIANGLES **Graph points *A*, *B*, and *C*. Connect the points to form △*ABC*. Decide whether △*ABC* is *acute, right,* or *obtuse*.**

27. $A(-2, 4)$, $B(6, 0)$, $C(-5, -2)$

28. $A(0, 2)$, $B(5, 1)$, $C(1, -1)$

29. **ALGEBRA** Tell whether a triangle with side lengths $5x$, $12x$, and $13x$ (where $x > 0$) is *acute*, *right*, or *obtuse*.

USING DIAGRAMS In Exercises 30 and 31, copy and complete the statement with <, >, or =, if possible. If it is not possible, *explain* why.



30. $m\angle A$ _?_ $m\angle D$

31. $m\angle B + m\angle C$ _?_ $m\angle E + m\angle F$

32. ★ **OPEN-ENDED MATH** The side lengths of a triangle are 6, 8, and x (where $x > 0$). What are the values of x that make the triangle a right triangle? an acute triangle? an obtuse triangle?

33. **ALGEBRA** The sides of a triangle have lengths x, $x + 4$, and 20. If the length of the longest side is 20, what values of x make the triangle acute?

34. **CHALLENGE** The sides of a triangle have lengths $4x + 6$, $2x + 1$, and $6x - 1$. If the length of the longest side is $6x - 1$, what values of x make the triangle obtuse?

PROBLEM SOLVING

EXAMPLE 3
on p. 443
for Ex. 35

35. **PAINTING** You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are 90°?

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36. **WALKING** You walk 749 feet due east to the gym from your home. From the gym you walk 800 feet southwest to the library. Finally, you walk 305 feet from the library back home. Do you live directly north of the library? *Explain.*

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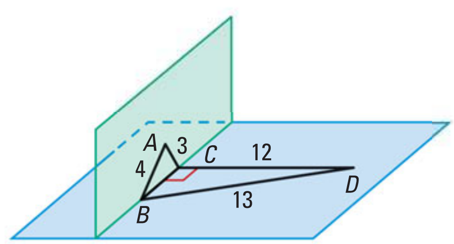
37. **MULTI-STEP PROBLEM** Use the diagram shown.



a. Find BC.

b. Use the Converse of the Pythagorean Theorem to show that $\triangle ABC$ is a right triangle.

c. Draw and label a similar diagram where $\triangle DBC$ remains a right triangle, but $\triangle ABC$ is not.

38. ★ **SHORT RESPONSE** You are setting up a volleyball net. To stabilize the pole, you tie one end of a rope to the pole 7 feet from the ground. You tie the other end of the rope to a stake that is 4 feet from the pole. The rope between the pole and stake is about 8 feet 4 inches long. Is the pole perpendicular to the ground? *Explain*. If it is not, how can you fix it?

39. ★ **EXTENDED RESPONSE** You are considering buying a used car. You would like to know whether the frame is sound. A sound frame of the car should be rectangular, so it has four right angles. You plan to measure the shadow of the car on the ground as the sun shines directly on the car.

a. You make a triangle with three tape measures on one corner. It has side lengths 12 inches, 16 inches, and 20 inches. Is this a right triangle? *Explain*.

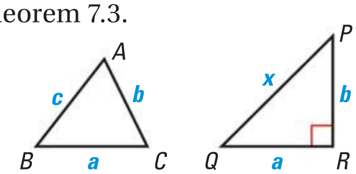
b. You make a triangle on a second corner with side lengths 9 inches, 12 inches, and 18 inches. Is this a right triangle? *Explain*.

c. The car owner says the car was never in an accident. Do you believe this claim? *Explain*.

40. **PROVING THEOREM 7.3** Copy and complete the proof of Theorem 7.3.

GIVEN ▶ In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side.

PROVE ▶ $\triangle ABC$ is an acute triangle.



Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x, where $\angle R$ is a right angle and x is the length of the longest side. Compare lengths c and x.

STATEMENTS	REASONS
1. In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	**1.** _?_
2. $a^2 + b^2 = x^2$	**2.** _?_
3. $c^2 < x^2$	**3.** _?_
4. $c < x$	**4.** A property of square roots
5. $m\angle R = 90°$	**5.** _?_
6. $m\angle C < m\angle$ _?_	**6.** Converse of the Hinge Theorem
7. $m\angle C < 90°$	**7.** _?_
8. $\angle C$ is an acute angle.	**8.** _?_
9. $\triangle ABC$ is an acute triangle.	**9.** _?_

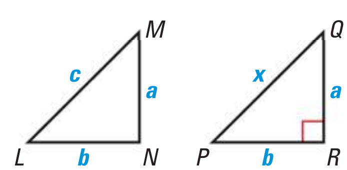
41. **PROVING THEOREM 7.4** Prove Theorem 7.4. Include a diagram and GIVEN and PROVE statements. (*Hint*: Look back at Exercise 40.)

42. **PROVING THEOREM 7.2** Prove the Converse of the Pythagorean Theorem.

GIVEN ▶ In $\triangle LMN$, $\overline{LM}$ is the longest side, and $c^2 = a^2 + b^2$.

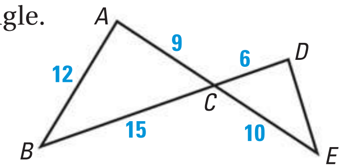
PROVE ▶ $\triangle LMN$ is a right triangle.

Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x. Compare lengths c and x.



★ = STANDARDIZED TEST PRACTICE

43. ★ **SHORT RESPONSE** *Explain* why $\angle D$ must be a right angle.



44. **COORDINATE PLANE** Use graph paper.

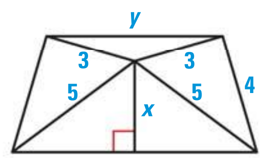
a. Graph $\triangle ABC$ with $A(-7, 2)$, $B(0, 1)$ and $C(-4, 4)$.

b. Use the slopes of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

c. Use the lengths of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

d. Did you get the same answer in parts (b) and (c)? If not, *explain* why.

45. **CHALLENGE** Find the values of x and y.

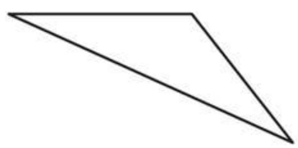


MIXED REVIEW

PREVIEW
Prepare for Lesson 7.3 in Exs. 46–48.

In Exercises 46–48, copy the triangle and draw one of its altitudes. *(p. 319)*

46.



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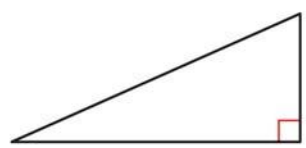

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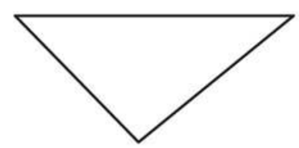


Copy and complete the statement. *(p. 364)*

49. If $\frac{10}{x} = \frac{7}{y}$, then $\frac{10}{7} = \frac{?}{?}$.

50. If $\frac{x}{15} = \frac{y}{2}$, then $\frac{x}{y} = \frac{?}{?}$.

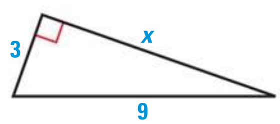
51. If $\frac{x}{8} = \frac{y}{9}$, then $\frac{x+8}{8} = \frac{?}{?}$.

52. The perimeter of a rectangle is 135 feet. The ratio of the length to the width is 8 : 1. Find the length and the width. *(p. 372)*

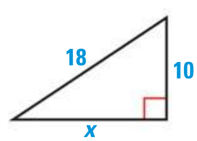
QUIZ *for Lessons 7.1–7.2*

Find the unknown side length. Write your answer in simplest radical form. *(p. 433)*

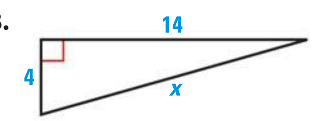
1.



2.



3.



Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*. *(p. 441)*

4. 6, 7, and 9

5. 10, 12, and 16

6. 8, 16, and $8\sqrt{6}$

7. 20, 21, and 29

8. 8, 3, $\sqrt{73}$

9. 8, 10, and 12

Investigating Geometry **ACTIVITY** *Use before Lesson 7.3*

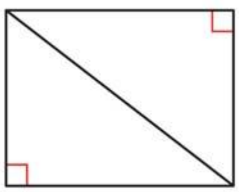
7.3 Similar Right Triangles

MATERIALS • rectangular piece of paper • ruler • scissors • colored pencils

QUESTION **How are geometric means related to the altitude of a right triangle?**

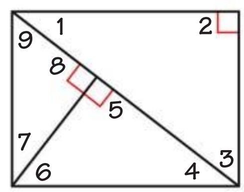
EXPLORE **Compare right triangles**

STEP 1



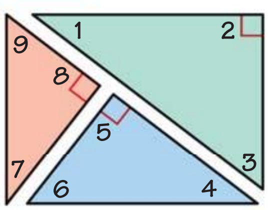
Draw a diagonal Draw a diagonal on your rectangular piece of paper to form two congruent right triangles.

STEP 2



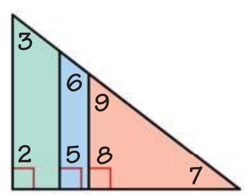
Draw an altitude Fold the paper to make an altitude to the hypotenuse of one of the triangles.

STEP 3



Cut and label triangles Cut the rectangle into the three right triangles that you drew. Label the angles and color the triangles as shown.

STEP 4



Arrange the triangles Arrange the triangles so ∠1, ∠4, and ∠7 are on top of each other as shown.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

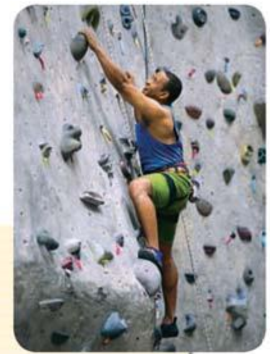
1. How are the two smaller right triangles related to the large triangle?

2. *Explain* how you would show that the green triangle is similar to the red triangle.

3. *Explain* how you would show that the red triangle is similar to the blue triangle.

4. The *geometric mean* of a and b is x if $\frac{a}{x} = \frac{x}{b}$. Write a proportion involving the side lengths of two of your triangles so that one side length is the geometric mean of the other two lengths in the proportion.

7.3 Use Similar Right Triangles



Before You identified the altitudes of a triangle.

Now You will use properties of the altitude of a right triangle.

Why? So you can determine the height of a wall, as in Example 4.

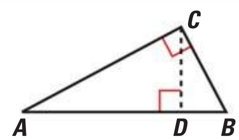
Key Vocabulary
- **altitude of a triangle,** *p. 320*
- **geometric mean,** *p. 359*
- **similar polygons,** *p. 372*

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

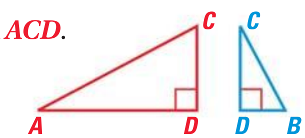
THEOREM — *For Your Notebook*

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



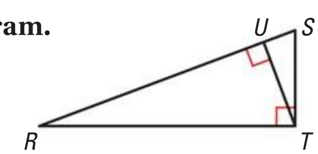
Proof: below; Ex. 35, p. 456

Plan for Proof of Theorem 7.5 First prove that $\triangle CBD \sim \triangle ABC$. Each triangle has a right angle and each triangle includes $\angle B$. The triangles are similar by the AA Similarity Postulate. Use similar reasoning to show that $\triangle ACD \sim \triangle ABC$.

To show $\angle CBD \sim \triangle ACD$, begin by showing $\angle ACD \cong \angle B$ because they are both complementary to $\angle DCB$. Each triangle also has a right angle, so you can use the AA Similarity Postulate.

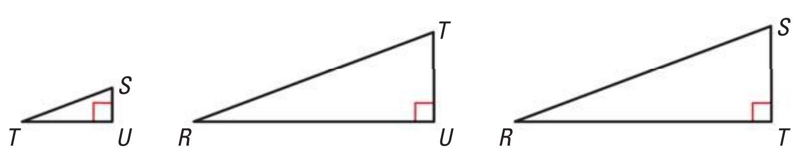
EXAMPLE 1 Identify similar triangles

Identify the similar triangles in the diagram.



Solution

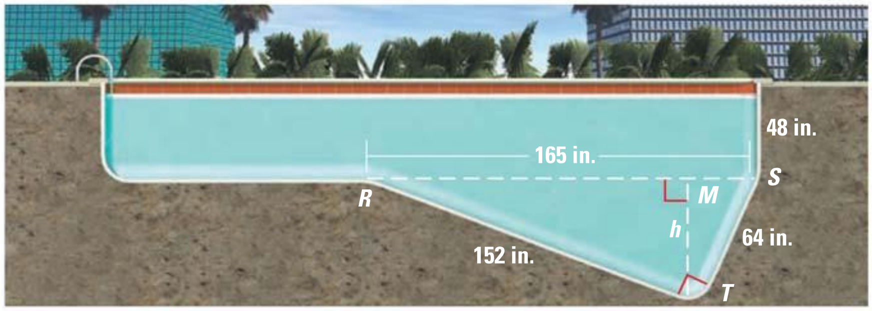
Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



▶ $\triangle TSU \sim \triangle RTU \sim \triangle RST$

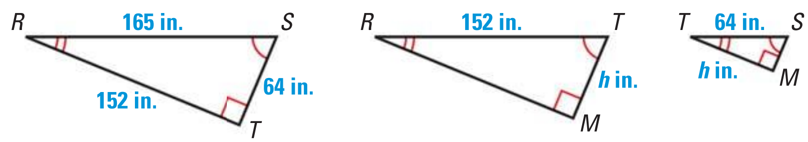
EXAMPLE 2 Find the length of the altitude to the hypotenuse

SWIMMING POOL The diagram below shows a cross-section of a swimming pool. What is the maximum depth of the pool?



Solution

STEP 1 **Identify** the similar triangles and sketch them.



$\triangle RST \sim \triangle RTM \sim \triangle TSM$

STEP 2 **Find** the value of h. Use the fact that $\triangle RST \sim \triangle RTM$ to write a proportion.

$\frac{TM}{ST} = \frac{TR}{SR}$	Corresponding side lengths of similar triangles are in proportion.
$\frac{h}{64} = \frac{152}{165}$	Substitute.
$165h = 64(152)$	Cross Products Property
$h \approx 59$	Solve for h.

STEP 3 **Read** the diagram above. You can see that the maximum depth of the pool is $h + 48$, which is about $59 + 48 = 107$ inches.

▶ The maximum depth of the pool is about 107 inches.

Animated Geometry at classzone.com

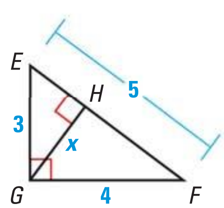
AVOID ERRORS

Notice that if you tried to write a proportion using $\triangle RTM$ and $\triangle TSM$, there would be two unknowns, so you would not be able to solve for h.

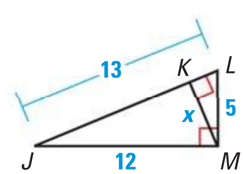
✓ GUIDED PRACTICE for Examples 1 and 2

Identify the similar triangles. Then find the value of x.

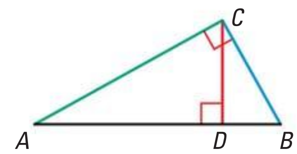
1.



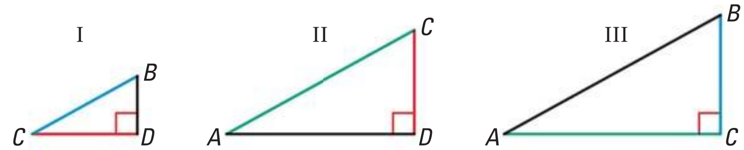
2.



GEOMETRIC MEANS In Lesson 6.1, you learned that the *geometric mean* of two numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Consider right $\triangle ABC$. From Theorem 7.5, you know that altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.



READ SYMBOLS
Remember that an altitude is defined as a segment. So, $\overline{CD}$ refers to an altitude in $\triangle ABC$ and CD refers to its length.



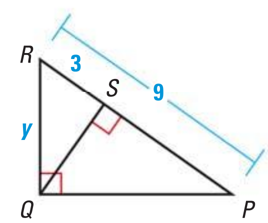
Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the geometric mean of BD and AD. As you see below, CB and AC are also geometric means of segment lengths in the diagram.

Proportions Involving Geometric Means in Right $\triangle ABC$

length of shorter leg of I / length of shorter leg of II	→	$\frac{BD}{CD} = \frac{CD}{AD}$	←	length of longer leg of I / length of longer leg of II
length of hypotenuse of III / length of hypotenuse of I	→	$\frac{AB}{CB} = \frac{CB}{DB}$	←	length of shorter leg of III / length of shorter leg of I
length of hypotenuse of III / length of hypotenuse of II	→	$\frac{AB}{AC} = \frac{AC}{AD}$	←	length of longer leg of III / length of longer leg of II

EXAMPLE 3 Use a geometric mean

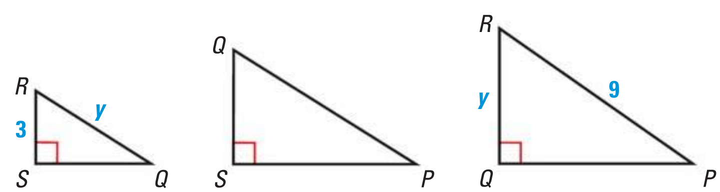
Find the value of y. Write your answer in simplest radical form.



Solution

REVIEW SIMILARITY
Notice that $\triangle RQS$ and $\triangle RPQ$ both contain the side with length y, so these are the similar pair of triangles to use to solve for y.

STEP 1 **Draw** the three similar triangles.



STEP 2 **Write** a proportion.

$$\frac{\text{length of hyp. of } \triangle RPQ}{\text{length of hyp. of } \triangle RQS} = \frac{\text{length of shorter leg of } \triangle RPQ}{\text{length of shorter leg of } \triangle RQS}$$

$\frac{9}{y} = \frac{y}{3}$ **Substitute.**

$27 = y^2$ **Cross Products Property**

$\sqrt{27} = y$ **Take the positive square root of each side.**

$3\sqrt{3} = y$ **Simplify.**

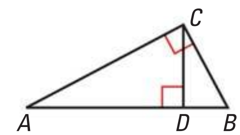
WRITE PROOFS
In Exercise 32 on page 455, you will use the geometric mean theorems to prove the Pythagorean Theorem.

THEOREMS *For Your Notebook*

THEOREM 7.6 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.



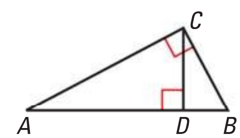
$$\frac{BD}{CD} = \frac{CD}{AD}$$

Proof: Ex. 36, p. 456

THEOREM 7.7 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



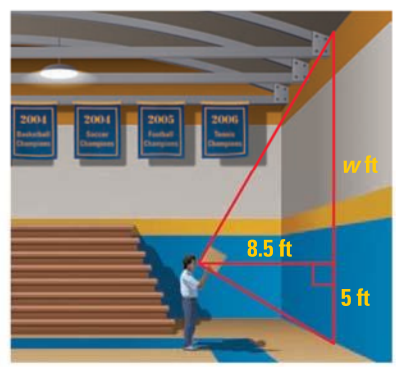
$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

Proof: Ex. 37, p. 456

EXAMPLE 4 Find a height using indirect measurement

ROCK CLIMBING WALL To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall.

You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the distance from you to the gym wall. Approximate the height of the gym wall.



Solution

By Theorem 7.6, you know that 8.5 is the geometric mean of w and 5.

$\frac{w}{8.5} = \frac{8.5}{5}$ **Write a proportion.**

$w \approx 14.5$ **Solve for *w*.**

▶ So, the height of the wall is $5 + w \approx 5 + 14.5 = 19.5$ feet.

✓ GUIDED PRACTICE for Examples 3 and 4

3. In Example 3, which theorem did you use to solve for y? *Explain.*

4. Mary is 5.5 feet tall. How far from the wall in Example 4 would she have to stand in order to measure its height?

7.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 20, 31, and 34

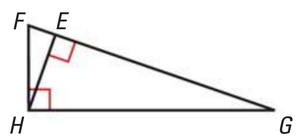
SKILL PRACTICE

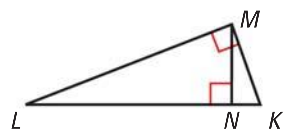
1. **VOCABULARY** Copy and complete: Two triangles are _?_ if their corresponding angles are congruent and their corresponding side lengths are proportional.

2. ★ **WRITING** In your own words, explain *geometric mean*.

EXAMPLE 1 on p. 449 for Exs. 3–4

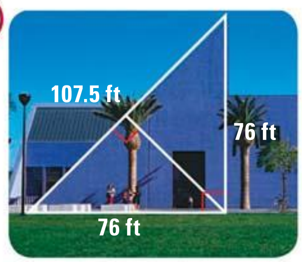
IDENTIFYING SIMILAR TRIANGLES **Identify the three similar right triangles in the given diagram.**

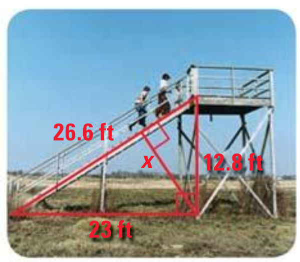
3. 

4. 

EXAMPLE 2 on p. 450 for Exs. 5–7

FINDING ALTITUDES **Find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

5. 

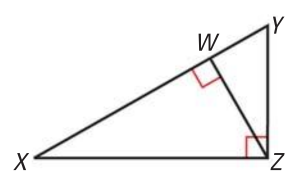
6. 

7. 

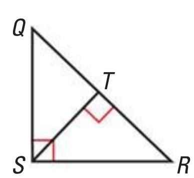
EXAMPLES 3 and 4 on pp. 451–452 for Exs. 8–18

COMPLETING PROPORTIONS **Write a similarity statement for the three similar triangles in the diagram. Then complete the proportion.**

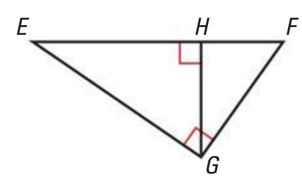
8. $\frac{XW}{?} = \frac{ZW}{YW}$



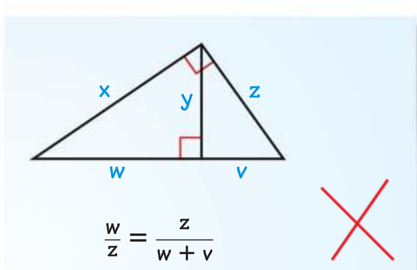
9. $\frac{?}{SQ} = \frac{SQ}{TQ}$

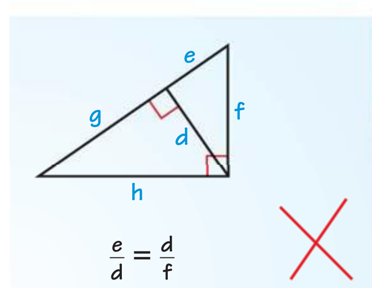


10. $\frac{EF}{EG} = \frac{EG}{?}$



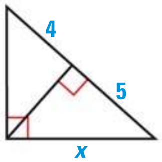
ERROR ANALYSIS ***Describe*** **and correct the error in writing a proportion for the given diagram.**

11. 

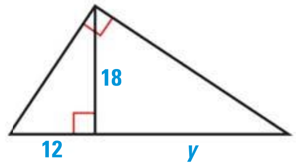
12. 

FINDING LENGTHS **Find the value of the variable. Round decimal answers to the nearest tenth.**

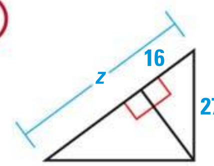
13.



14.



15.



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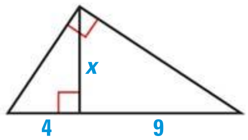
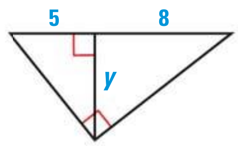


17.



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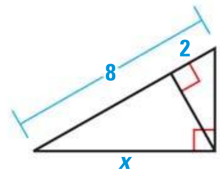
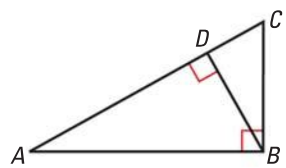


19. ★ **MULTIPLE CHOICE** Use the diagram at the right. Decide which proportion is false.

Ⓐ $\frac{DB}{DC} = \frac{DA}{DB}$ Ⓑ $\frac{CA}{AB} = \frac{AB}{AD}$

Ⓒ $\frac{CA}{BA} = \frac{BA}{CA}$ Ⓓ $\frac{DC}{BC} = \frac{BC}{CA}$

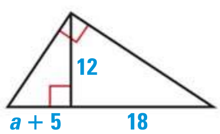


20. ★ **MULTIPLE CHOICE** In the diagram in Exercise 19 above, $AC = 36$ and $BC = 18$. Find AD. If necessary, round to the nearest tenth.

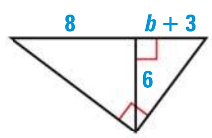
Ⓐ 9 Ⓑ 15.6 Ⓒ 27 Ⓓ 31.2

xy ALGEBRA **Find the value(s) of the variable(s).**

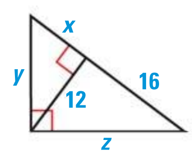
21.



22.

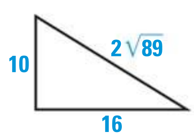


23.

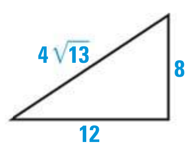


USING THEOREMS **Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

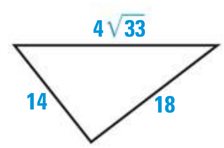
24.



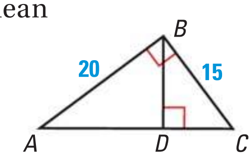
25.



26.



27. **FINDING LENGTHS** Use the Geometric Mean Theorems to find AC and BD.



28. **CHALLENGE** Draw a right isosceles triangle and label the two leg lengths x. Then draw the altitude to the hypotenuse and label its length y. Now draw the three similar triangles and label any side length that is equal to either x or y. What can you conclude about the relationship between the two smaller triangles? *Explain.*

PROBLEM SOLVING

29. **DOGHOUSE** The peak of the doghouse shown forms a right angle. Use the given dimensions to find the height of the roof.

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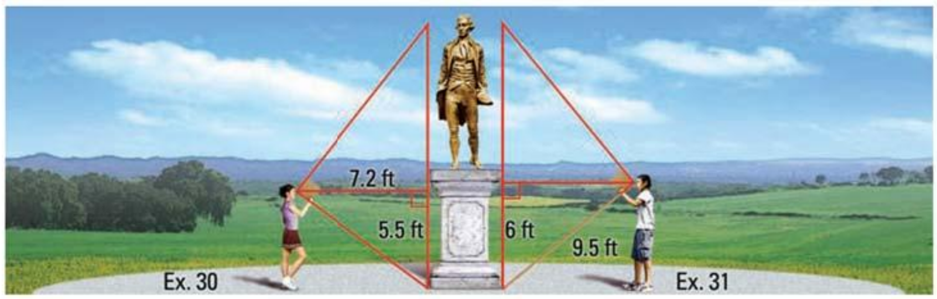
EXAMPLE 4 on p. 452 for Exs. 30–31

30. **MONUMENT** You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument. Mary measures the vertical distance from the ground to your eye and the distance from you to the monument. Approximate the height of the monument (as shown at the left below).



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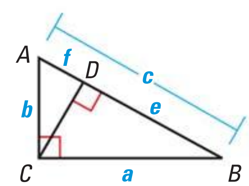
31. ★ **SHORT RESPONSE** Paul is standing on the other side of the monument in Exercise 30 (as shown at the right above). He has a piece of rope staked at the base of the monument. He extends the rope to the cardboard square he is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 30? *Explain*.

32. **PROVING THEOREM 7.1** Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem.

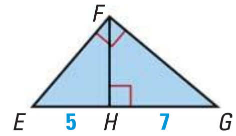


GIVEN ▶ In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE ▶ $c^2 = a^2 + b^2$

STATEMENTS	REASONS
1. Draw $\triangle ABC$. $\angle BCA$ is a right angle.	**1.** ?
2. Draw a perpendicular from C to $\overline{AB}$.	**2.** Perpendicular Postulate
3. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	**3.** ?
4. $ce = a^2$ and $cf = b^2$	**4.** ?
5. $ce + b^2 =$? $+ b^2$	**5.** Addition Property of Equality
6. $ce + cf = a^2 + b^2$	**6.** ?
7. $c(e + f) = a^2 + b^2$	**7.** ?
8. $e + f =$?	**8.** Segment Addition Postulate
9. $c \cdot c = a^2 + b^2$	**9.** ?
10. $c^2 = a^2 + b^2$	**10.** Simplify.

33. **MULTI-STEP PROBLEM** Use the diagram.

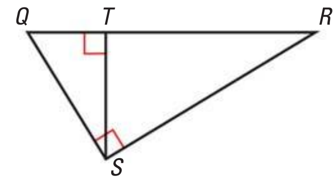


a. Name all the altitudes in $\triangle EGF$. *Explain.*

b. Find *FH*.

c. Find the area of the triangle.

34. ★ **EXTENDED RESPONSE** Use the diagram.

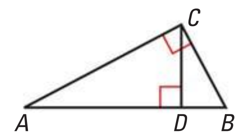


a. Sketch the three similar triangles in the diagram. Label the vertices. *Explain* how you know which vertices correspond.

b. Write similarity statements for the three triangles.

c. Which segment's length is the geometric mean of *RT* and *RQ*? *Explain* your reasoning.

PROVING THEOREMS **In Exercises 35–37, use the diagram and GIVEN statements below.**



GIVEN ▶ $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

35. Prove Theorem 7.5 by using the Plan for Proof on page 449.

36. Prove Theorem 7.6 by showing $\frac{BD}{CD} = \frac{CD}{AD}$.

37. Prove Theorem 7.7 by showing $\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$.

38. **CHALLENGE** The *harmonic mean* of *a* and *b* is $\frac{2ab}{a + b}$. The Greek mathematician Pythagoras found that three equally taut strings on stringed instruments will sound harmonious if the length of the middle string is equal to the harmonic mean of the lengths of the shortest and longest string.

a. Find the harmonic mean of 10 and 15.

b. Find the harmonic mean of 6 and 14.

c. Will equally taut strings whose lengths have the ratio 4 : 6 : 12 sound harmonious? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.4 in Exs. 39–46.

Simplify the expression. *(p. 874)*

39. $\sqrt{27} \cdot \sqrt{2}$ **40.** $\sqrt{8} \cdot \sqrt{10}$ **41.** $\sqrt{12} \cdot \sqrt{7}$ **42.** $\sqrt{18} \cdot \sqrt{12}$

43. $\frac{5}{\sqrt{7}}$ **44.** $\frac{8}{\sqrt{11}}$ **45.** $\frac{15}{\sqrt{27}}$ **46.** $\frac{12}{\sqrt{24}}$

Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer. *(p. 171)*

47. **Line 1:** (2, 4), (4, 2)
Line 2: (3, 5), (−1, 1)

48. **Line 1:** (0, 2), (−1, −1)
Line 2: (3, 1), (1, −5)

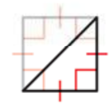
49: **Line 1:** (1, 7), (4, 7)
Line 2: (5, 2), (7, 4)

7.4 Special Right Triangles

Before	You found side lengths using the Pythagorean Theorem.
Now	You will use the relationships among the sides in special right triangles.
Why?	So you can find the height of a drawbridge, as in Ex. 28.

Key Vocabulary
- **isosceles triangle,** *p. 217*

A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half as shown.



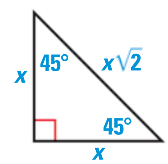
THEOREM *For Your Notebook*

THEOREM 7.8 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$

Proof: Ex. 30, p. 463

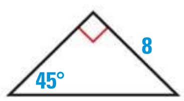


USE RATIOS
The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

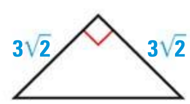
EXAMPLE 1 Find hypotenuse length in a 45°-45°-90° triangle

Find the length of the hypotenuse.

a.



b.



Solution

a. By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle, so by Theorem 7.8, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$ — 45°-45°-90° Triangle Theorem

= $8\sqrt{2}$ — Substitute.

b. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$ — 45°-45°-90° Triangle Theorem

= $3\sqrt{2} \cdot \sqrt{2}$ — Substitute.

= $3 \cdot 2$ — Product of square roots

= 6 — Simplify.

REVIEW ALGEBRA
Remember the following properties of radicals:

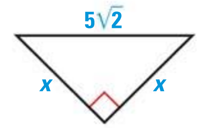
$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$

$\sqrt{a \cdot a} = a$

For a review of radical expressions, see p. 874.

EXAMPLE 2 Find leg lengths in a 45°-45°-90° triangle

Find the lengths of the legs in the triangle.



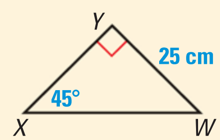
Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$	45°-45°-90° Triangle Theorem
$5\sqrt{2} = x \cdot \sqrt{2}$	Substitute.
$\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$	Divide each side by $\sqrt{2}$.
$5 = x$	Simplify.

EXAMPLE 3 Standardized Test Practice

Triangle *WXY* is a right triangle. Find the length of $\overline{WX}$.



Ⓐ 50 cm Ⓑ $25\sqrt{2}$ cm

Ⓒ 25 cm Ⓓ $\frac{25\sqrt{2}}{2}$ cm

ELIMINATE CHOICES
You can eliminate choices C and D because the hypotenuse has to be longer than the leg.

Solution

By the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

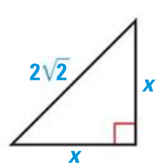
$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$	45°-45°-90° Triangle Theorem
$WX = 25\sqrt{2}$	Substitute.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

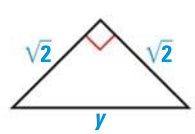
GUIDED PRACTICE for Examples 1, 2, and 3

Find the value of the variable.

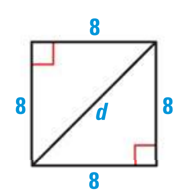
1.



2.



3.



4. Find the leg length of a 45°-45°-90° triangle with a hypotenuse length of 6.

A 30°-60°-90° triangle can be formed by dividing an equilateral triangle in half.

THEOREM *For Your Notebook*

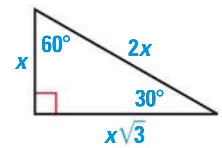
THEOREM 7.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse $= 2 \cdot$ shorter leg

longer leg $=$ shorter leg $\cdot \sqrt{3}$

Proof: Ex. 32, p. 463



USE RATIOS
The extended ratio of the side lengths of a 30°-60°-90° triangle is $1 : \sqrt{3} : 2$.

EXAMPLE 4 Find the height of an equilateral triangle

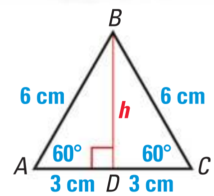
LOGO The logo on the recycling bin at the right resembles an equilateral triangle with side lengths of 6 centimeters. What is the approximate height of the logo?

Solution

Draw the equilateral triangle described. Its altitude forms the longer leg of two 30°-60°-90° triangles. The length h of the altitude is approximately the height of the logo.

longer leg $=$ shorter leg $\cdot \sqrt{3}$

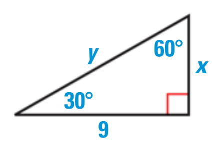
$h = 3 \cdot \sqrt{3} \approx 5.2$ cm



REVIEW MEDIAN
Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ bisects $\overline{AC}$.

EXAMPLE 5 Find lengths in a 30°-60°-90° triangle

xy **Find the values of x and y. Write your answer in simplest radical form.**



STEP 1 **Find** the value of x.

longer leg $=$ shorter leg $\cdot \sqrt{3}$	**30°-60°-90° Triangle Theorem**
$9 = x\sqrt{3}$	**Substitute.**
$\frac{9}{\sqrt{3}} = x$	**Divide each side by $\sqrt{3}$.**
$\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$	**Multiply numerator and denominator by $\sqrt{3}$.**
$\frac{9\sqrt{3}}{3} = x$	**Multiply fractions.**
$3\sqrt{3} = x$	**Simplify.**

STEP 2 **Find** the value of y.

hypotenuse $= 2 \cdot$ shorter leg	**30°-60°-90° Triangle Theorem**
$y = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$	**Substitute and simplify.**

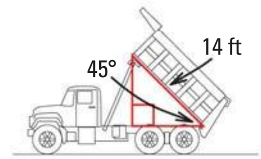
EXAMPLE 6 Find a height

DUMP TRUCK The body of a dump truck is raised to empty a load of sand. How high is the 14 foot body from the frame when it is tipped upward at the given angle?

a. $45°$ angle **b.** $60°$ angle

Solution

a. When the body is raised $45°$ above the frame, the height h is the length of a leg of a $45°$-$45°$-$90°$ triangle. The length of the hypotenuse is 14 feet.



$14 = h \cdot \sqrt{2}$ — **45°-45°-90° Triangle Theorem**

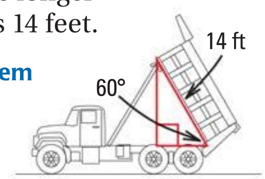
$\frac{14}{\sqrt{2}} = h$ — **Divide each side by $\sqrt{2}$.**

$9.9 \approx h$ — **Use a calculator to approximate.**

REWRITE MEASURES
To write 9.9 ft in feet and inches, multiply the decimal part by 12.
$12 \cdot 0.9 = 10.8$
So, 9.9 ft is about 9 feet 11 inches.

▶ When the angle of elevation is $45°$, the body is about 9 feet 11 inches above the frame.

b. When the body is raised $60°$, the height h is the length of the longer leg of a $30°$-$60°$-$90°$ triangle. The length of the hypotenuse is 14 feet.



hypotenuse $= 2 \cdot$ shorter leg — **30°-60°-90° Triangle Theorem**

$14 = 2 \cdot s$ — **Substitute.**

$7 = s$ — **Divide each side by 2.**

longer leg $=$ shorter leg $\cdot \sqrt{3}$ — **30°-60°-90° Triangle Theorem**

$h = 7\sqrt{3}$ — **Substitute.**

$h \approx 12.1$ — **Use a calculator to approximate.**

▶ When the angle of elevation is $60°$, the body is about 12 feet 1 inch above the frame.

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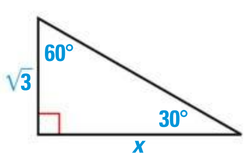
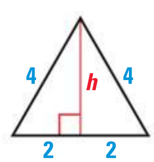
✓ GUIDED PRACTICE for Examples 4, 5, and 6

Find the value of the variable.

5.



6.



7. WHAT IF? In Example 6, what is the height of the body of the dump truck if it is raised $30°$ above the frame?

8. In a $30°$-$60°$-$90°$ triangle, *describe* the location of the shorter side. *Describe* the location of the longer side?

7.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 19, 22, 29, and 34

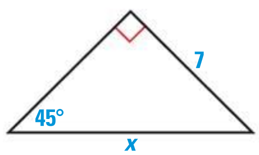
SKILL PRACTICE

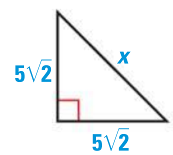
1. **VOCABULARY** Copy and complete: A triangle with two congruent sides and a right angle is called _?_.

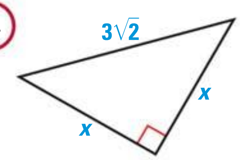
2. ★ **WRITING** *Explain* why the acute angles in an isosceles right triangle always measure 45°.

EXAMPLES 1 and 2 on pp. 457–458 for Exs. 3–5

45°-45°-90° TRIANGLES **Find the value of x. Write your answer in simplest radical form.**

3. 

4. 

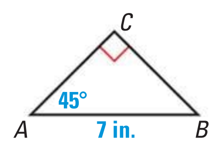
5. 

EXAMPLE 3 on p. 458 for Exs. 6–7

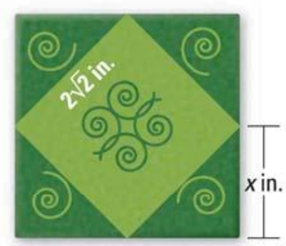
6. ★ **MULTIPLE CHOICE** Find the length of $\overline{AC}$.

Ⓐ $7\sqrt{2}$ in. Ⓑ $2\sqrt{7}$ in.

Ⓒ $\frac{7\sqrt{2}}{2}$ in. Ⓓ $\sqrt{14}$ in.

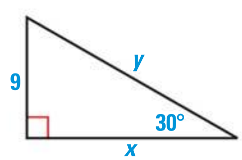


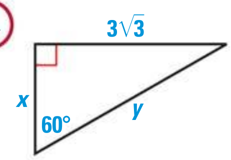
7. **ISOSCELES RIGHT TRIANGLE** The square tile shown has painted corners in the shape of congruent 45°-45°-90° triangles. What is the value of x? What is the side length of the tile?

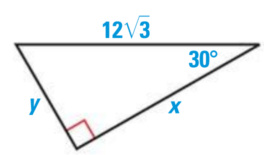


EXAMPLES 4 and 5 on p. 459 for Exs. 8–10

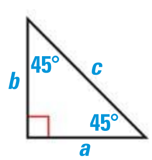
30°-60°-90° TRIANGLES **Find the value of each variable. Write your answers in simplest radical form.**

8. 

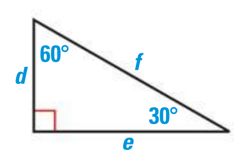
9. 

10. 

SPECIAL RIGHT TRIANGLES **Copy and complete the table.**

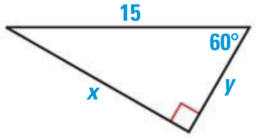
11. 

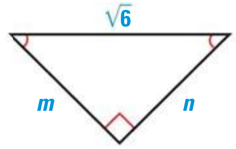
a	7	?	?	?	$\sqrt{5}$
b	?	11	?	?	?
c	?	?	10	$6\sqrt{2}$	?

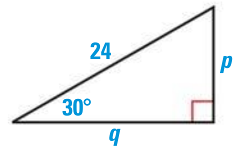
12. 

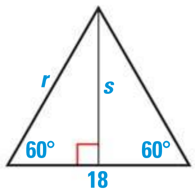
d	5	?	?	?	?
e	?	?	$8\sqrt{3}$	?	12
f	?	14	?	$18\sqrt{3}$	?

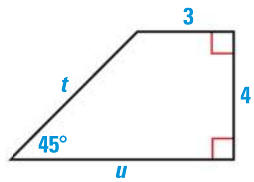
ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

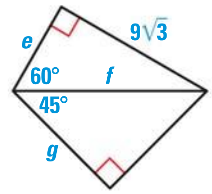
13. 

14. 

15. 

16. 

17. 

18. 

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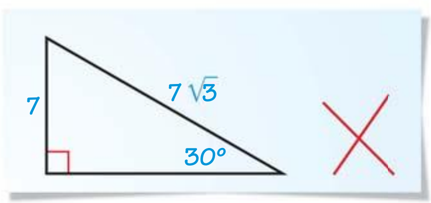
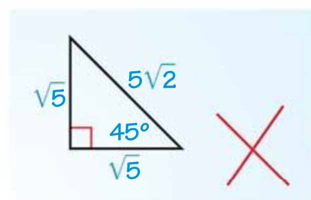
19. ★ **MULTIPLE CHOICE** Which side lengths do *not* represent a 30°-60°-90° triangle?

Ⓐ $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1$ Ⓑ $\sqrt{2}, \sqrt{6}, 2\sqrt{2}$

Ⓒ $\frac{5}{2}, \frac{5\sqrt{3}}{2}, 10$ Ⓓ $3, 3\sqrt{3}, 6$

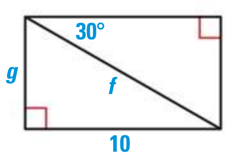
ERROR ANALYSIS ***Describe*** **and correct the error in finding the length of the hypotenuse.**

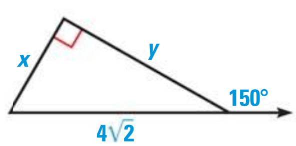
20. 

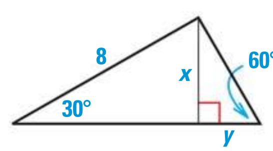
21. 

22. ★ **WRITING** Abigail solved Example 5 on page 459 in a different way. Instead of dividing each side by $\sqrt{3}$, she multiplied each side by $\sqrt{3}$. Does her method work? *Explain* why or why not.

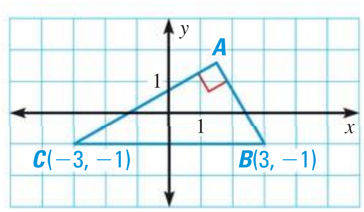
ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

23. 

24. 

25. 

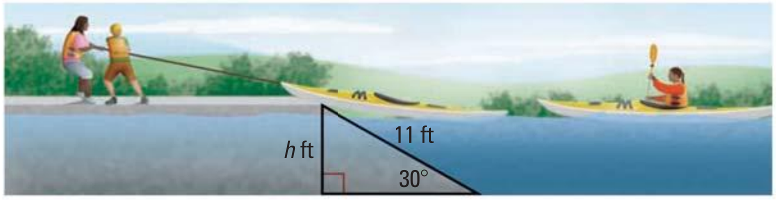
26. **CHALLENGE** $\triangle ABC$ is a 30°-60°-90° triangle. Find the coordinates of A.



PROBLEM SOLVING

EXAMPLE 6 on p. 460 for Ex. 27

27. KAYAK RAMP A ramp is used to launch a kayak. What is the height of an 11 foot ramp when its angle is 30° as shown?



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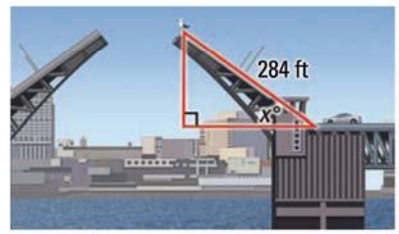
28. DRAWBRIDGE Each half of the drawbridge is about 284 feet long, as shown. How high does a seagull rise who is on the end of the drawbridge when the angle with measure $x°$ is 30°? 45°? 60°?



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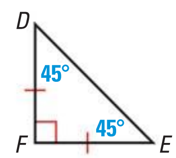
29. ★ SHORT RESPONSE *Describe* two ways to show that all isosceles right triangles are similar to each other.

30. PROVING THEOREM 7.8 Write a paragraph proof of the 45°-45°-90° Triangle Theorem.

GIVEN ▶ $\triangle DEF$ is a 45°-45°-90° triangle.

PROVE ▶ The hypotenuse is $\sqrt{2}$ times as long as each leg.

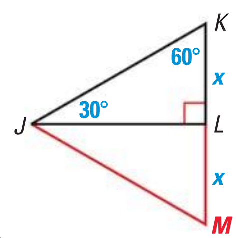


31. EQUILATERAL TRIANGLE If an equilateral triangle has a side length of 20 inches, find the height of the triangle.

32. PROVING THEOREM 7.9 Write a paragraph proof of the 30°-60°-90° Triangle Theorem.

GIVEN ▶ $\triangle JKL$ is a 30°-60°-90° triangle.

PROVE ▶ The hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



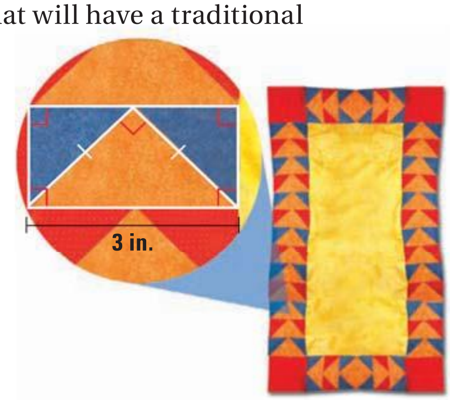
Plan for Proof Construct $\triangle JML$ congruent to $\triangle JKL$. Then prove that $\triangle JKM$ is equilateral. Express the lengths of $\overline{JK}$ and $\overline{JL}$ in terms of x.

33. MULTI-STEP PROBLEM You are creating a quilt that will have a traditional "flying geese" border, as shown below.

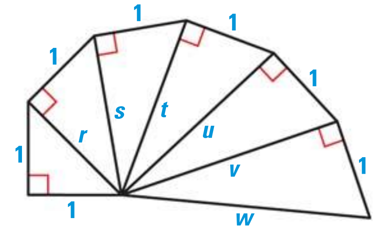
a. Find all the angle measures of the small blue triangles and the large orange triangles.

b. The width of the border is to be 3 inches. To create the large triangle, you cut a square of fabric in half. Not counting any extra fabric needed for seams, what size square do you need?

c. What size square do you need to create each small triangle?



34. ★ **EXTENDED RESPONSE** Use the figure at the right. You can use the fact that the converses of the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem are true.



a. Find the values of r, s, t, u, v, and w. *Explain* the procedure you used to find the values.

b. Which of the triangles, if any, is a 45°-45°-90° triangle? *Explain*.

c. Which of the triangles, if any, is a 30°-60°-90° triangle? *Explain*.

35. **CHALLENGE** In quadrilateral $QRST$, $m\angle R = 60°$, $m\angle T = 90°$, $QR = RS$, $ST = 8$, $TQ = 8$, and $\overline{RT}$ and $\overline{QS}$ intersect at point Z.

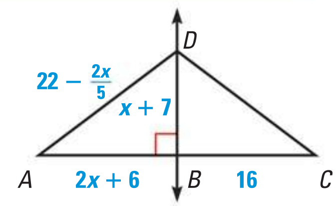
a. Draw a diagram.

b. *Explain* why $\triangle RQT \cong \triangle RST$.

c. Which is longer, QS or RT? *Explain*.

MIXED REVIEW

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$. *(p. 303)*



36. Which pairs of segment lengths are equal?

37. What is the value of x?

38. Find CD.

Is it possible to build a triangle using the given side lengths? *(p. 328)*

39. 4, 4, and 7 **40.** 3, 3, and $9\sqrt{2}$ **41.** 7, 15, and 21

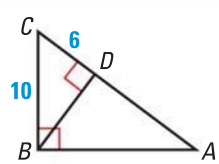
PREVIEW
Prepare for Lesson 7.5 in Exs. 42–44.

Tell whether the given side lengths form a right triangle. *(p. 441)*

42. 21, 22, and $5\sqrt{37}$ **43.** $\frac{3}{2}$, 2, and $\frac{5}{2}$ **44.** 8, 10, and 14

QUIZ for Lessons 7.3–7.4

In Exercises 1 and 2, use the diagram. *(p. 449)*

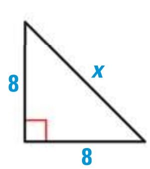


1. Which segment's length is the geometric mean of AC and CD?

2. Find BD, AD, and AB.

Find the values of the variable(s). Write your answer(s) in simplest radical form. *(p. 457)*

3.



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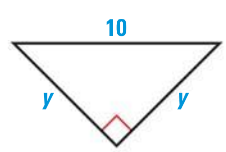
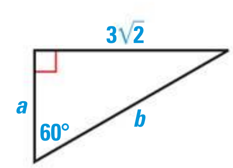


5.



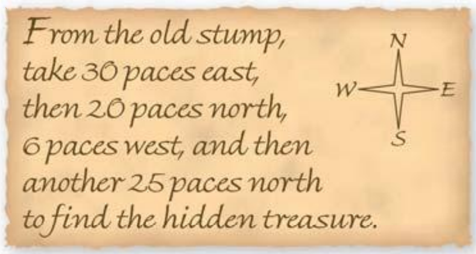
 EXTRA PRACTICE for Lesson 7.4, p. 908 **ONLINE QUIZ** at classzone.com

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
classzone.com

Lessons 7.1–7.4

1. **GRIDDED ANSWER** Find the direct distance, in paces, from the treasure to the stump.



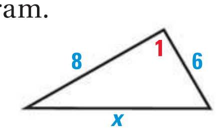
2. **MULTI-STEP PROBLEM** On a map of the United States, you put a pushpin on three state capitols you want to visit: Jefferson City, Missouri; Little Rock, Arkansas; and Atlanta, Georgia.

   a. Draw a diagram to model the triangle.

   b. Do the pushpins form a right triangle? If not, what type of triangle do they form?

3. **SHORT RESPONSE** Bob and John started running at 10 A.M. Bob ran east at 4 miles per hour while John ran south at 5 miles per hour. How far apart were they at 11:30 A.M.? *Describe* how you calculated the answer.

4. **EXTENDED RESPONSE** Give all values of x that make the statement true for the given diagram.



   a. $\angle 1$ is a right angle. *Explain*.

   b. $\angle 1$ is an obtuse angle. *Explain*.

   c. $\angle 1$ is an acute angle. *Explain*.

   d. The triangle is isosceles. *Explain*.

   e. No triangle is possible. *Explain*.

5. **EXTENDED RESPONSE** A Chinese checker board is made of triangles. Use the picture below to answer the questions.

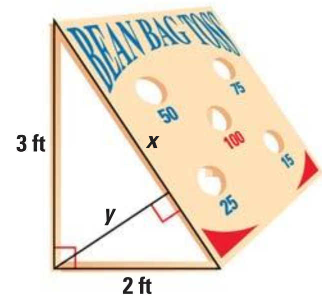
   a. Count the marble holes in the purple triangle. What kind of triangle is it?

   b. If a side of the purple triangle measures 8 centimeters, find the area of the purple triangle.

   c. How many marble holes are in the center hexagon? Assuming each marble hole takes up the same amount of space, what is the relationship between the purple triangle and center hexagon?

   d. Find the area of the center hexagon. *Explain* your reasoning.

6. **MULTI-STEP PROBLEM** You build a beanbag toss game. The game is constructed from a sheet of plywood supported by two boards. The two boards form a right angle and their lengths are 3 feet and 2 feet.



   a. Find the length x of the plywood.

   b. You put in a support that is the altitude y to the hypotenuse of the right triangle. What is the length of the support?

   c. Where does the support attach to the plywood? *Explain*.

7.5 Apply the Tangent Ratio

Before You used congruent or similar triangles for indirect measurement.

Now You will use the tangent ratio for indirect measurement.

Why? So you can find the height of a roller coaster, as in Ex. 32.

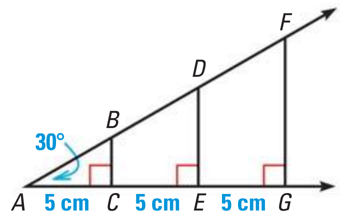
Key Vocabulary
- **trigonometric ratio**
- **tangent**

ACTIVITY RIGHT TRIANGLE RATIO

Materials: metric ruler, protractor, calculator

STEP 1 **Draw** a 30° angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.

STEP 2 **Measure** the legs of each right triangle. Copy and complete the table.

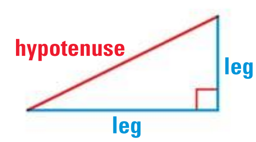


Triangle	Adjacent leg	Opposite leg	$\frac{\text{Opposite leg}}{\text{Adjacent leg}}$
$\triangle ABC$	5 cm	?	?
$\triangle ADE$	10 cm	?	?
$\triangle AFG$	15 cm	?	?

STEP 3 **Explain** why the proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true.

STEP 4 **Make** a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.



The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the **tangent** of the angle.

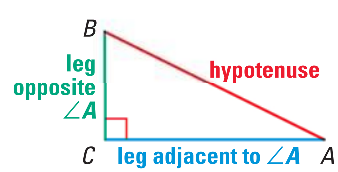
ABBREVIATE
Remember these abbreviations:
tangent → tan
opposite → opp.
adjacent → adj.

KEY CONCEPT *For Your Notebook*

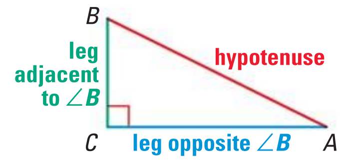
Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as $\tan A$) is defined as follows:

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

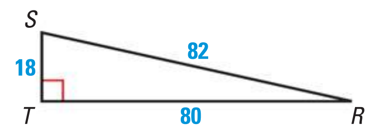


COMPLEMENTARY ANGLES In the right triangle, $\angle A$ and $\angle B$ are complementary so you can use the same diagram to find the tangent of $\angle A$ and the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg *opposite* $\angle B$ and the leg opposite $\angle A$ is the leg *adjacent* to $\angle B$.



EXAMPLE 1 Find tangent ratios

Find tan *S* and tan *R*. Write each answer as a fraction and as a decimal rounded to four places.



Solution

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

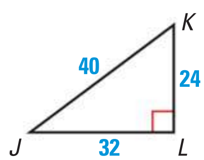
$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

APPROXIMATE
Unless told otherwise, you should round the values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

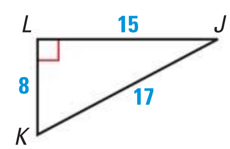
GUIDED PRACTICE for Example 1

Find tan *J* and tan *K*. Round to four decimal places.

1.

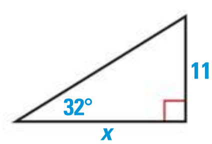


2.



EXAMPLE 2 Find a leg length

ALGEBRA **Find the value of *x*.**



Solution

Use the tangent of an acute angle to find a leg length.

$\tan 32° = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 32°.
$\tan 32° = \frac{11}{x}$	Substitute.
$x \cdot \tan 32° = 11$	Multiply each side by *x*.
$x = \frac{11}{\tan 32°}$	Divide each side by tan 32°.
$x \approx \frac{11}{0.6249}$	Use a calculator to find tan 32°.
$x \approx 17.6$	Simplify.

ANOTHER WAY
You can also use the Table of Trigonometric Ratios on p. 925 to find the decimal values of trigonometric ratios.

EXAMPLE 3 Estimate height using tangent

LAMPPOST Find the height h of the lamppost to the nearest inch.

$\tan 70° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 70°.

$\tan 70° = \frac{h}{40}$ Substitute.

$40 \cdot \tan 70° = h$ Multiply each side by 40.

$109.9 \approx h$ Use a calculator to simplify.

▶ The lamppost is about 110 inches tall.

SPECIAL RIGHT TRIANGLES You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

EXAMPLE 4 Use a special right triangle to find a tangent

Use a special right triangle to find the tangent of a 60° angle.

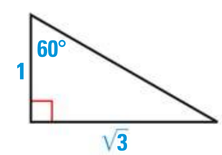
SIMILAR TRIANGLES
The tangents of all 60° angles are the same constant ratio. Any right triangle with a 60° angle can be used to determine this value.

STEP 1 **Because** all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.

longer leg = shorter leg $\cdot \sqrt{3}$ 30°-60°-90° Triangle Theorem

$x = 1 \cdot \sqrt{3}$ Substitute.

$x = \sqrt{3}$ Simplify.



STEP 2 **Find** tan 60°.

$\tan 60° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 60°.

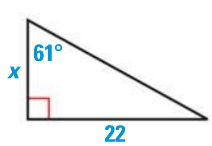
$\tan 60° = \frac{\sqrt{3}}{1}$ Substitute.

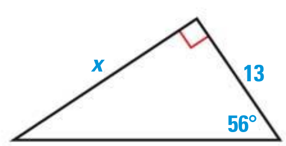
$\tan 60° = \sqrt{3}$ Simplify.

▶ The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

✓ **GUIDED PRACTICE** for Examples 2, 3, and 4

Find the value of x. Round to the nearest tenth.

3. 

4. 

5. **WHAT IF?** In Example 4, suppose the side length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$.

7.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 17, 35, and 37

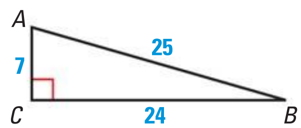
SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The tangent ratio compares the length of _?_ to the length of _?_.

2. ★ **WRITING** *Explain* how you know that all right triangles with an acute angle measuring $n°$ are similar to each other.

EXAMPLE 1 on p. 467 for Exs. 3–5

FINDING TANGENT RATIOS **Find tan A and tan B. Write each answer as a fraction and as a decimal rounded to four places.**

3.


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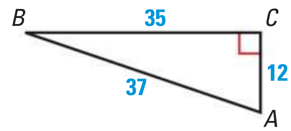

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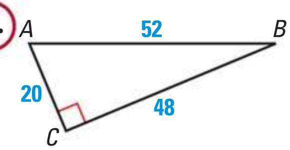
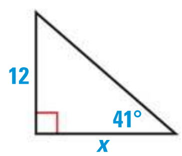


EXAMPLE 2 on p. 467 for Exs. 6–8

FINDING LEG LENGTHS **Find the value of x to the nearest tenth.**

6.


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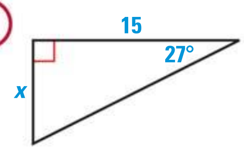

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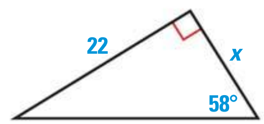
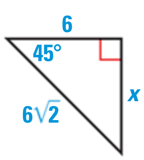


EXAMPLE 4 on p. 468 for Exs. 9–12

FINDING LEG LENGTHS **Find the value of x using the definition of tangent. Then find the value of x using the 45°-45°-90° Theorem or the 30°-60°-90° Theorem. *Compare* the results.**

9.


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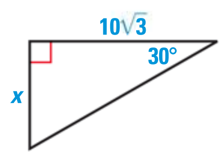

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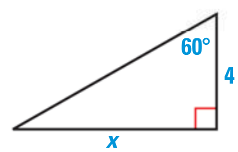
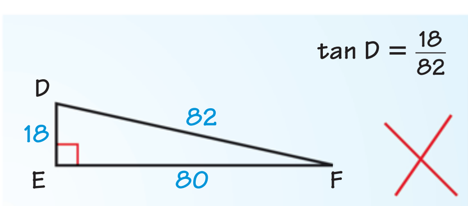
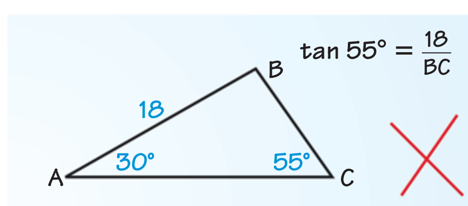


12. **SPECIAL RIGHT TRIANGLES** Find tan 30° and tan 45° using the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem.

ERROR ANALYSIS ***Describe*** **the error in the statement of the tangent ratio. Correct the statement, if possible. Otherwise, write *not possible*.**

13.


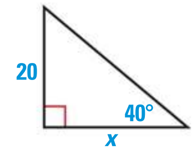
14.


15. ★ **WRITING** *Describe* what you must know about a triangle in order to use the tangent ratio.

16. ★ **MULTIPLE CHOICE** Which expression can be used to find the value of x in the triangle shown?

Ⓐ $x = 20 \cdot \tan 40°$ Ⓑ $x = \dfrac{\tan 40°}{20}$

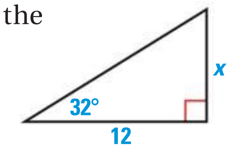
Ⓒ $x = \dfrac{20}{\tan 40°}$ Ⓓ $x = \dfrac{20}{\tan 50°}$



17. ★ **MULTIPLE CHOICE** What is the approximate value of x in the triangle shown?

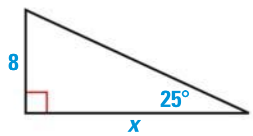
Ⓐ 0.4 Ⓑ 2.7

Ⓒ 7.5 Ⓓ 19.2



FINDING LEG LENGTHS **Use a tangent ratio to find the value of *x*. Round to the nearest tenth. Check your solution using the tangent of the other acute angle.**

18.



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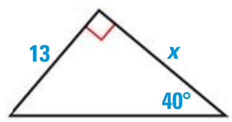

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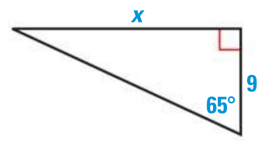
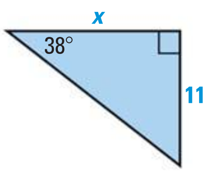


FINDING AREA **Find the area of the triangle. Round to the nearest tenth.**

21.



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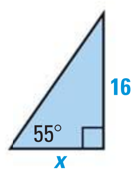

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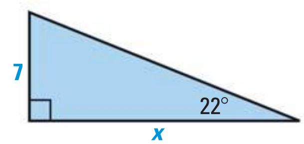
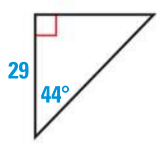


FINDING PERIMETER **Find the perimeter of the triangle. Round to the nearest tenth.**

24.



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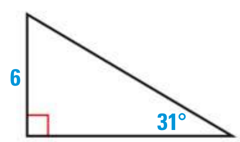

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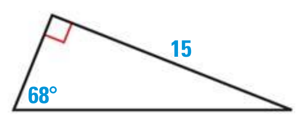
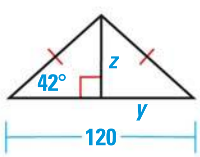


FINDING LENGTHS **Find *y*. Then find *z*. Round to the nearest tenth.**

27.



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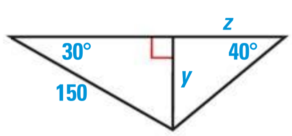

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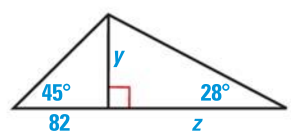
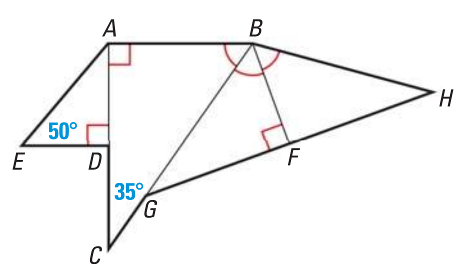


30. **CHALLENGE** Find the perimeter of the figure at the right, where $AC = 26$, $AD = BF$, and D is the midpoint of $\overline{AC}$.



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 3 on p. 468 for Exs. 31–32

31. WASHINGTON MONUMENT A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle between the ground and the top of the monument to be 78°. Find the height h of the Washington Monument to the nearest foot.

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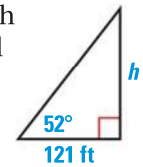
32. ROLLER COASTERS A roller coaster makes an angle of 52° with the ground. The horizontal distance from the crest of the hill to the bottom of the hill is about 121 feet, as shown. Find the height h of the roller coaster to the nearest foot.



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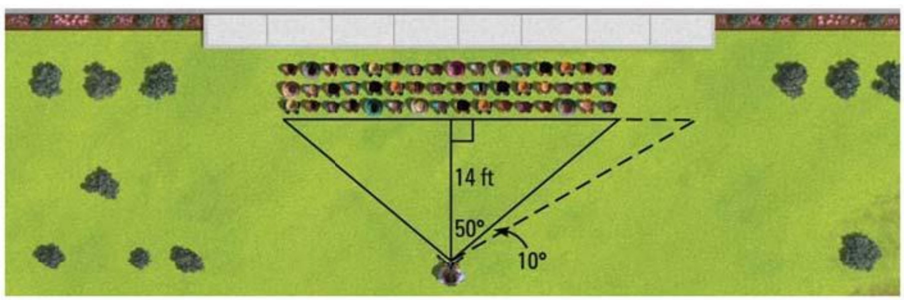
CLASS PICTURE **Use this information and diagram for Exercises 33 and 34.**

Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. If she looks toward either end of the class, she turns 50°.



33. ISOSCELES TRIANGLE What is the distance between the ends of the class?

34. MULTI-STEP PROBLEM The photographer wants to estimate how many more students can fit at the end of the first row. The photographer turns 50° to see the last student and another 10° to see the end of the camera range.

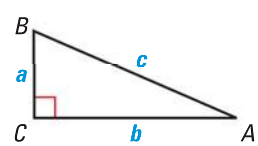
a. Find the distance from the center to the last student in the row.

b. Find the distance from the center to the end of the camera range.

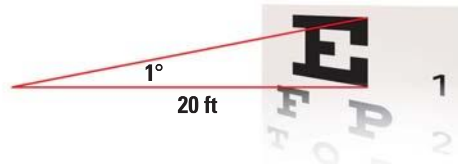
c. Use the results of parts (a) and (b) to estimate the length of the empty space.

d. If each student needs 2 feet of space, about how many more students can fit at the end of the first row? *Explain* your reasoning.

35. ★ SHORT RESPONSE Write expressions for the tangent of each acute angle in the triangle. *Explain* how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair are $\angle A$ and $\angle B$?

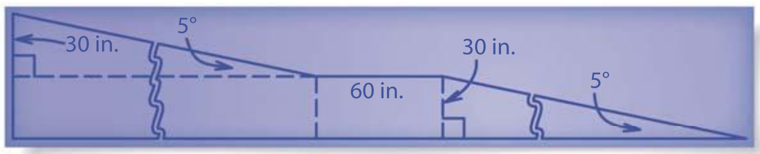


36. **EYE CHART** You are looking at an eye chart that is 20 feet away. Your eyes are level with the bottom of the "E" on the chart. To see the top of the "E," you look up 1°. How tall is the "E"?



Not drawn to scale

37. ★ **EXTENDED RESPONSE** According to the Americans with Disabilities Act, a ramp cannot have an incline that is greater than 5°. The regulations also state that the maximum rise of a ramp is 30 inches. When a ramp needs to reach a height greater than 30 inches, a series of ramps connected by 60 inch landings can be used, as shown below.

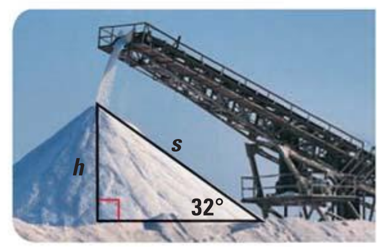


a. What is the maximum horizontal length of the base of one ramp, in feet? Round to the nearest foot.

b. If a doorway is 7.5 feet above the ground, what is the least number of ramps and landings you will need to lead to the doorway? Draw and label a diagram to *justify* your answer.

c. To the nearest foot, what is the total length of the base of the system of ramps and landings in part (b)?

38. **CHALLENGE** The road salt shown is stored in a cone-shaped pile. The base of the cone has a circumference of 80 feet. The cone rises at an angle of 32°. Find the height h of the cone. Then find the length s of the cone-shaped pile.



Mixed Review

The expressions given represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

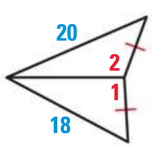
39. $m\angle A = x°$
$m\angle B = 4x°$
$m\angle C = 4x°$

40. $m\angle A = x°$
$m\angle B = x°$
$m\angle C = (5x - 60)°$

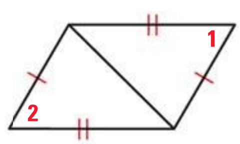
41. $m\angle A = (x + 20)°$
$m\angle B = (3x + 15)°$
$m\angle C = (x - 30)°$

Copy and complete the statement with <, >, or =. *Explain.* *(p. 335)*

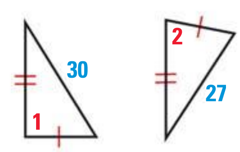
42. $m\angle 1$ __?__ $m\angle 2$



43. $m\angle 1$ __?__ $m\angle 2$



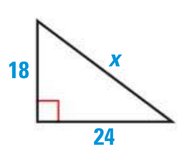
44. $m\angle 1$ __?__ $m\angle 2$



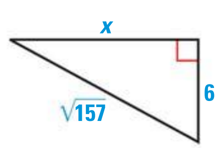
PREVIEW
Prepare for Lesson 7.6 in Exs. 45–47.

Find the unknown side length of the right triangle. *(p. 433)*

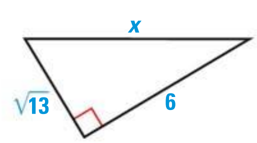
45.



46.



47.



7.6 Apply the Sine and Cosine Ratios

Before You used the tangent ratio.

Now You will use the sine and cosine ratios.

Why So you can find distances, as in Ex. 39.

Key Vocabulary
- **sine**
- **cosine**
- **angle of elevation**
- **angle of depression**

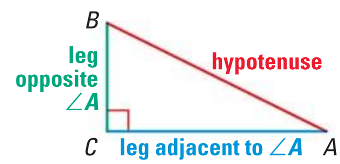
The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

ABBREVIATE
Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

KEY CONCEPT — *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

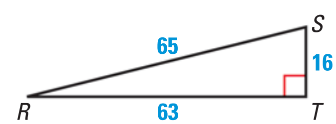


$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

EXAMPLE 1 Find sine ratios

Find $\sin S$ and $\sin R$. Write each answer as a fraction and as a decimal rounded to four places.



Solution

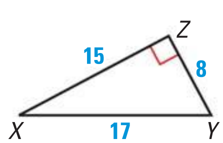
$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

$$\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

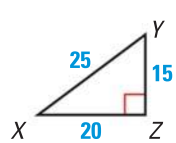
✓ GUIDED PRACTICE for Example 1

Find $\sin X$ and $\sin Y$. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

1.

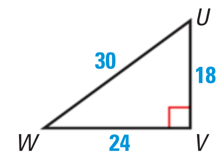


2.



EXAMPLE 2 Find cosine ratios

Find cos U and cos W. Write each answer as a fraction and as a decimal.



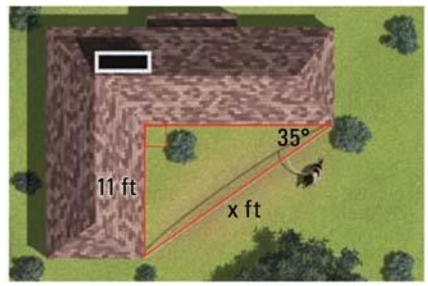
Solution

$$\cos U = \frac{\text{adj. to } \angle U}{\text{hyp.}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000$$

$$\cos W = \frac{\text{adj. to } \angle W}{\text{hyp.}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000$$

EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse

DOG RUN You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need.



Solution

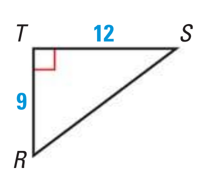
$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$	Write ratio for sine of 35°.
$\sin 35° = \frac{11}{x}$	Substitute.
$x \cdot \sin 35° = 11$	Multiply each side by x.
$x = \frac{11}{\sin 35°}$	Divide each side by sin 35°.
$x \approx \frac{11}{0.5736}$	Use a calculator to find sin 35°.
$x \approx 19.2$	Simplify.

▶ You will need a little more than 19 feet of cable.

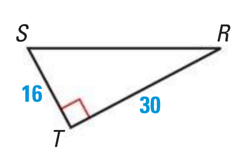
✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, find cos R and cos S. Write each answer as a decimal. Round to four decimal places, if necessary.

3.



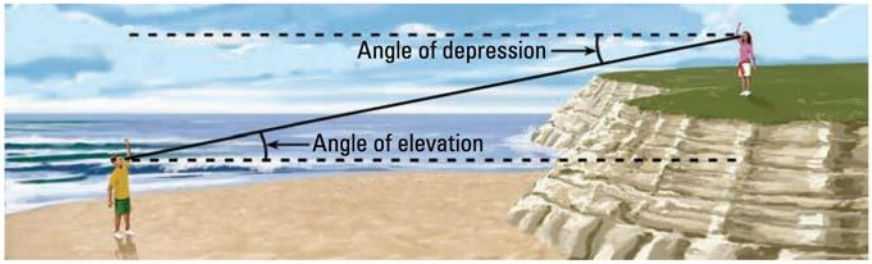
4.



5. In Example 3, use the cosine ratio to find the length of the other leg of the triangle formed.

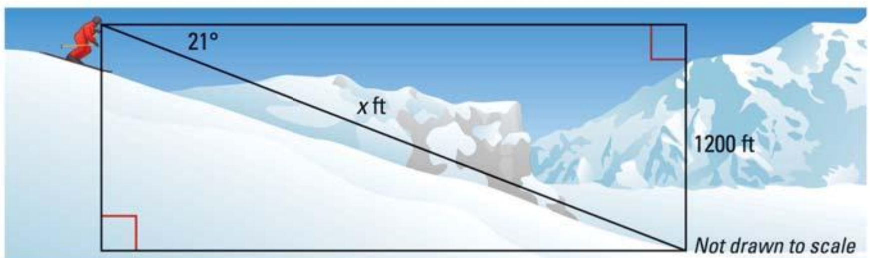
ANGLES If you look up at an object, the angle your line of sight makes with a horizontal line is called the **angle of elevation**. If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of depression**.

APPLY THEOREMS
Notice that the angle of elevation and the angle of depression are congruent by the Alternate Interior Angles Theorem on page 155.



EXAMPLE 4 Find a hypotenuse using an angle of depression

SKIING You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?



Solution

$\sin 21° = \frac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 21°.**

$\sin 21° = \frac{1200}{x}$ **Substitute.**

$x \cdot \sin 21° = 1200$ **Multiply each side by x.**

$x = \frac{1200}{\sin 21°}$ **Divide each side by sin 21°.**

$x \approx \frac{1200}{0.3584}$ **Use a calculator to find sin 21°.**

$x \approx 3348.2$ **Simplify.**

▶ You ski about 3348 meters down the mountain.

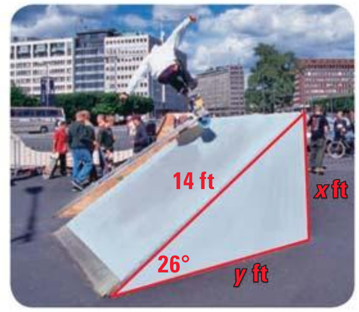
AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Example 4

6. **WHAT IF?** Suppose the angle of depression in Example 4 is 28°. About how far would you ski?

EXAMPLE 5 Find leg lengths using an angle of elevation

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and length of the base of the ramp.



ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 481 for the **Problem Solving Workshop**.

Solution

STEP 1 **Find** the height.

$\sin 26° = \frac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 26°.**

$\sin 26° = \frac{x}{14}$ **Substitute.**

$14 \cdot \sin 26° = x$ **Multiply each side by 14.**

$6.1 \approx x$ **Use a calculator to simplify.**

▶ The height is about 6.1 feet.

STEP 2 **Find** the length of the base.

$\cos 26° = \frac{\text{adj.}}{\text{hyp.}}$ **Write ratio for cosine of 26°.**

$\cos 26° = \frac{y}{14}$ **Substitute.**

$14 \cdot \cos 26° = y$ **Multiply each side by 14.**

$12.6 \approx y$ **Use a calculator to simplify.**

▶ The length of the base is about 12.6 feet.

EXAMPLE 6 Use a special right triangle to find a sine and cosine

Use a special right triangle to find the sine and cosine of a 60° angle.

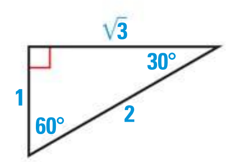
DRAW DIAGRAMS
As in Example 4 on page 468, to simplify calculations you can choose 1 as the length of the shorter leg.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 60° angle.

$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$

$\cos 60° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$



✓ GUIDED PRACTICE for Examples 5 and 6

7. **WHAT IF?** In Example 5, suppose the angle of elevation is 35°. What is the new height and base length of the ramp?

8. Use a special right triangle to find the sine and cosine of a 30° angle.

7.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 29, 35, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

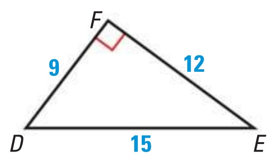
SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The sine ratio compares the length of _?_ to the length of _?_.

2. ★ **WRITING** *Explain* how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse.

EXAMPLE 1 on p. 473 for Exs. 3–6

FINDING SINE RATIOS **Find sin *D* and sin *E*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

3.


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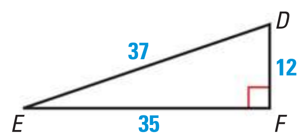
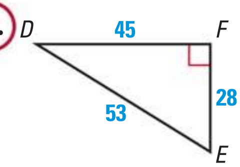
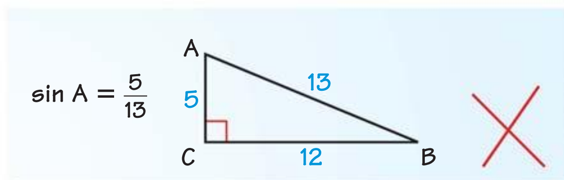


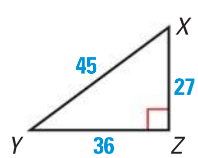
5.


6. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the sine of the angle.



EXAMPLE 2 on p. 474 for Exs. 7–9

FINDING COSINE RATIOS **Find cos *X* and cos *Y*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

7.


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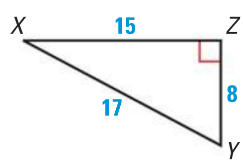

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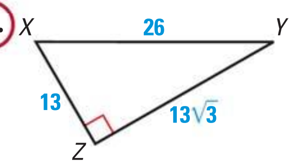
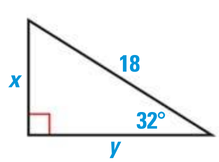


EXAMPLE 3 on p. 474 for Exs. 10–15

USING SINE AND COSINE RATIOS **Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.**

10.


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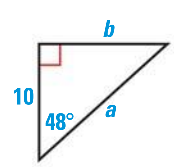

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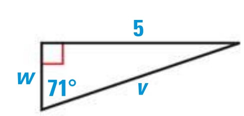

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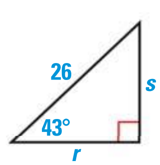

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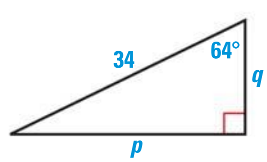
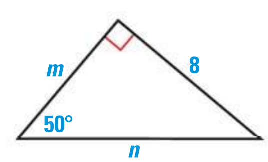


15.


EXAMPLE 6 on p. 476 for Ex. 16

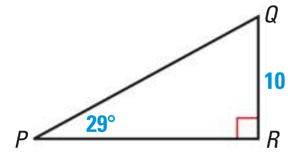
16. **SPECIAL RIGHT TRIANGLES** Use the 45°-45°-90° Triangle Theorem to find the sine and cosine of a 45° angle.

17. ★ **WRITING** *Describe* what you must know about a triangle in order to use the sine ratio and the cosine ratio.

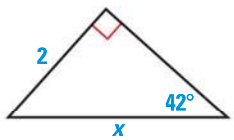
18. ★ **MULTIPLE CHOICE** In $\triangle PQR$, which expression can be used to find PQ?

Ⓐ $10 \cdot \cos 29°$ Ⓑ $10 \cdot \sin 29°$

Ⓒ $\dfrac{10}{\sin 29°}$ Ⓓ $\dfrac{10}{\cos 29°}$



ALGEBRA **Find the value of *x*. Round decimals to the nearest tenth.**

19.


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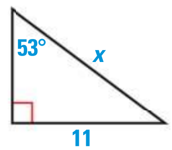
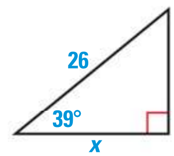


21.


FINDING SINE AND COSINE RATIOS **Find the unknown side length. Then find sin *X* and cos *X*. Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.**

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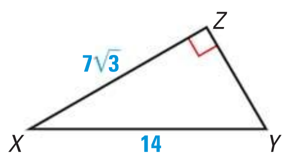

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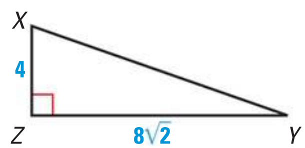

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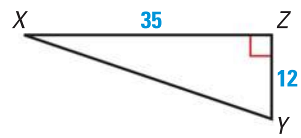

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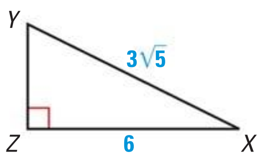

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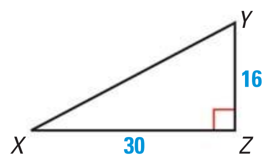

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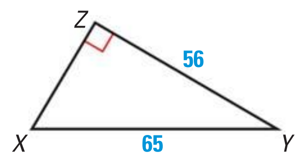


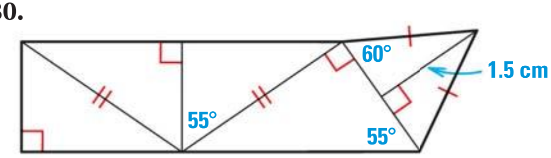
28. **ANGLE MEASURE** Make a prediction about how you could use trigonometric ratios to find angle measures in a triangle.

29. ★ **MULTIPLE CHOICE** In $\triangle JKL$, $m\angle L = 90°$. Which statement about $\triangle JKL$ *cannot* be true?

Ⓐ $\sin J = 0.5$ Ⓑ $\sin J = 0.1071$

Ⓒ $\sin J = 0.8660$ Ⓓ $\sin J = 1.1$

PERIMETER **Find the approximate perimeter of the figure.**

30.


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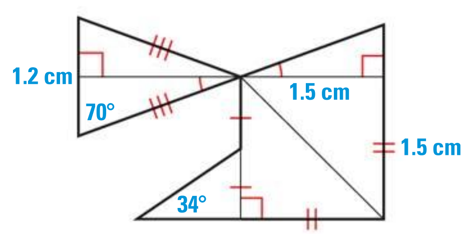

32. **CHALLENGE** Let A be any acute angle of a right triangle. Show that (a) $\tan A = \dfrac{\sin A}{\cos A}$ and (b) $(\sin A)^2 + (\cos A)^2 = 1$.

PROBLEM SOLVING

EXAMPLES 4 and 5 on pp. 475–476 for Exs. 33–36

33. AIRPLANE RAMP The airplane door is 19 feet off the ground and the ramp has a 31° angle of elevation. What is the length y of the ramp?

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34. BLEACHERS Find the horizontal distance h the bleachers cover. Round to the nearest foot.

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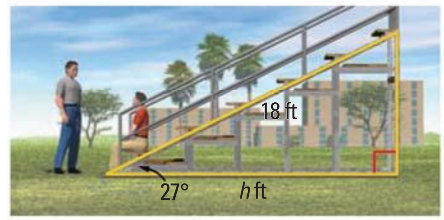


35. ★ **SHORT RESPONSE** You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 41°.

a. Draw and label a diagram to represent the situation.

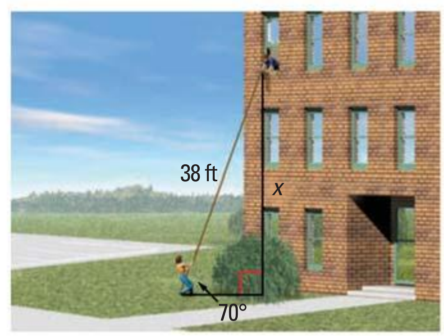
b. How far off the ground is the kite if you hold the spool 5 feet off the ground? *Describe* how the height where you hold the spool affects the height of the kite.

36. MULTI-STEP PROBLEM You want to hang a banner that is 29 feet tall from the third floor of your school. You need to know how tall the wall is, but there is a large bush in your way.

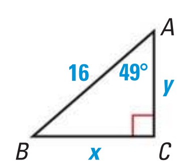
a. You throw a 38 foot rope out of the window to your friend. She extends it to the end and measures the angle of elevation to be 70°. How high is the window?

b. The bush is 6 feet tall. Will your banner fit above the bush?

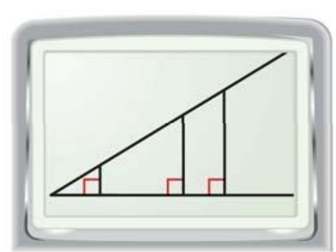
c. What If? Suppose you need to find how far from the school your friend needs to stand. Which trigonometric ratio should you use?



37. ★ **SHORT RESPONSE** Nick uses the equation $\sin 49° = \frac{x}{16}$ to find BC in $\triangle ABC$. Tim uses the equation $\cos 41° = \frac{x}{16}$. Which equation produces the correct answer? *Explain*.



38. TECHNOLOGY Use geometry drawing software to construct an angle. Mark three points on one side of the angle and construct segments perpendicular to that side at the points. Measure the legs of each triangle and calculate the sine of the angle. Is the sine the same for each triangle?



39. **MULTIPLE REPRESENTATIONS** You are standing on a cliff 30 feet above an ocean. You see a sailboat on the ocean.

a. Drawing a Diagram Draw and label a diagram of the situation.

b. Making a Table Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.

c. Drawing a Graph Graph the values you found in part (b), with the angle measures on the x-axis.

d. Making a Prediction Predict the length of the line of sight when the angle of depression is 30°.

40. **ALGEBRA** If $\triangle EQU$ is equilateral and $\triangle RGT$ is a right triangle with $RG = 2$, $RT = 1$, and $m\angle T = 90°$, show that $\sin E = \cos G$.

41. **CHALLENGE** Make a conjecture about the relationship between sine and cosine values.

a. Make a table that gives the sine and cosine values for the acute angles of a 45°-45°-90° triangle, a 30°-60°-90° triangle, a 34°-56°-90° triangle, and a 17°-73°-90° triangle.

b. Compare the sine and cosine values. What pattern(s) do you notice?

c. Make a conjecture about the sine and cosine values in part (b).

d. Is the conjecture in part (c) true for right triangles that are not special right triangles? *Explain.*

MIXED REVIEW

Rewrite the equation so that x is a function of y. *(p. 877)*

42. $y = \sqrt{x}$

43. $y = 3x - 10$

44. $y = \frac{x}{9}$

PREVIEW
Prepare for Lesson 7.7 in Exs. 45–47.

Copy and complete the table. *(p. 884)*

45.

x	$\sqrt{x}$
?	0
?	1
?	$\sqrt{2}$
?	2
?	4

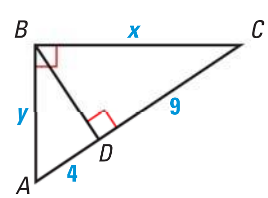
46.

x	$\frac{1}{x}$
?	1
?	$\frac{1}{2}$
?	3
?	$\frac{2}{7}$
?	7

47.

x	$\frac{2}{7}x + 4$
?	0
?	2
?	6
?	8
?	10

48. Find the values of x and y in the triangle at the right. *(p. 449)*



PROBLEM SOLVING WORKSHOP
LESSON 7.6

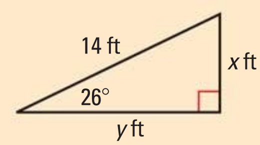
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 476

MULTIPLE REPRESENTATIONS You can use the Pythagorean Theorem, tangent ratio, sine ratio, or cosine ratio to find the length of an unknown side of a right triangle. The decision of which method to use depends upon what information you have. In some cases, you can use more than one method to find the unknown length.

PROBLEM

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and base of the ramp.



METHOD 1

Using a Cosine Ratio and the Pythagorean Theorem

STEP 1 **Find** the measure of the third angle.

$26° + 90° + m\angle 3 = 180°$ — **Triangle Sum Theorem**

$116° + m\angle 3 = 180°$ — **Combine like terms.**

$m\angle 3 = 64°$ — **Subtract 116° from each side.**

STEP 2 **Use** the cosine ratio to find the height of the ramp.

$\cos 64° = \frac{\text{adj.}}{\text{hyp.}}$ — **Write ratio for cosine of 64°.**

$\cos 64° = \frac{x}{14}$ — **Substitute.**

$14 \cdot \cos 64° = x$ — **Multiply each side by 14.**

$6.1 \approx x$ — **Use a calculator to simplify.**

▶ The height is about 6.1 feet.

STEP 3 **Use** the Pythagorean Theorem to find the length of the base of the ramp.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ — **Pythagorean Theorem**

$14^2 = 6.1^2 + y^2$ — **Substitute.**

$196 = 37.21 + y^2$ — **Multiply.**

$158.79 = y^2$ — **Subtract 37.21 from each side.**

$12.6 \approx y$ — **Find the positive square root.**

▶ The length of the base is about 12.6 feet.

METHOD 2 **Using a Tangent Ratio**

Use the tangent ratio and $h = 6.1$ feet to find the length of the base of the ramp.

$$\tan 26^\circ = \frac{\text{opp.}}{\text{adj.}}$$ Write ratio for tangent of 26°.

$$\tan 26^\circ = \frac{6.1}{y}$$ Substitute.

$$y \cdot \tan 26^\circ = 61$$ Multiply each side by *y*.

$$y = \frac{6.1}{\tan 26^\circ}$$ Divide each side by tan 26°.

$$y \approx 12.5$$ Use a calculator to simplify.

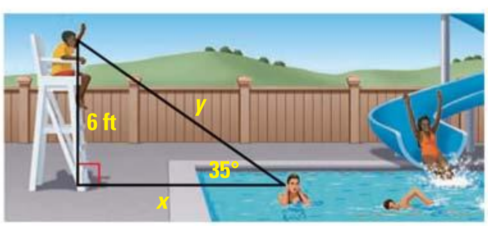
▶ The length of the base is about 12.5 feet.

Notice that when using the Pythagorean Theorem, the length of the base is 12.6 feet, but when using the tangent ratio, the length of the base is 12.5 feet. The tenth of a foot difference is due to the rounding error introduced when finding the height of the ramp and using that rounded value to calculate the length of the base.

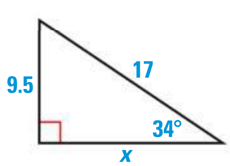
PRACTICE

1. **WHAT IF?** Suppose the length of the skateboard ramp is 20 feet. Find the height and base of the ramp.

2. **SWIMMER** The angle of elevation from the swimmer to the lifeguard is 35°. Find the distance x from the swimmer to the base of the lifeguard chair. Find the distance y from the swimmer to the lifeguard.

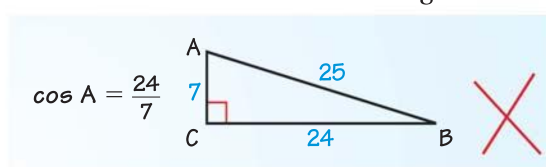


3. **xy ALGEBRA** Use the triangle below to write three different equations you can use to find the unknown leg length.



4. **SHORT RESPONSE** *Describe* how you would decide whether to use the Pythagorean Theorem or trigonometric ratios to find the lengths of unknown sides of a right triangle.

5. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the cosine of the angle.



6. **EXTENDED RESPONSE** You want to find the height of a tree in your yard. The tree's shadow is 15 feet long and you measure the angle of elevation from the end of the shadow to the top of tree to be 75°.
   a. Find the height of the tree. *Explain* the method you chose to solve the problem.
   b. What else would you need to know to solve this problem using similar triangles.
   c. *Explain* why you cannot use the sine ratio to find the height of the tree.

7.7 Solve Right Triangles

Before You used tangent, sine, and cosine ratios.

Now You will use inverse tangent, sine, and cosine ratios.

Why? So you can build a saddlerack, as in Ex. 39.

Key Vocabulary
- **solve a right triangle**
- **inverse tangent**
- **inverse sine**
- **inverse cosine**

To **solve a right triangle** means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:

- Two side lengths
- One side length and the measure of one acute angle

In Lessons 7.5 and 7.6, you learned how to use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. Once you know the tangent, the sine, or the cosine of an acute angle, you can use a calculator to find the measure of the angle.

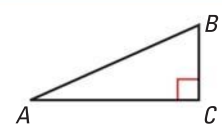
READ VOCABULARY

The expression "$\tan^{-1} x$" is read as "the inverse tangent of x."

KEY CONCEPT *For Your Notebook*

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



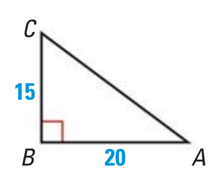
Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\cos^{-1} \frac{AC}{AB} = m\angle A$

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Solution

Because $\tan A = \frac{15}{20} = \frac{3}{4} = 0.75$, $\tan^{-1} 0.75 = m\angle A$. Use a calculator.

$\tan^{-1} 0.75 \approx 36.86989765 \cdots$

▶ So, the measure of $\angle A$ is approximately 36.9°.

EXAMPLE 2 Use an inverse sine and an inverse cosine

ANOTHER WAY

You can use the Table of Trigonometric Ratios on p. 925 to approximate $\sin^{-1} 0.87$ to the nearest degree. Find the number closest to 0.87 in the sine column and read the angle measure at the left.

Let $\angle A$ and $\angle B$ be acute angles in a right triangle. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

a. $\sin A = 0.87$ **b.** $\cos B = 0.15$

Solution

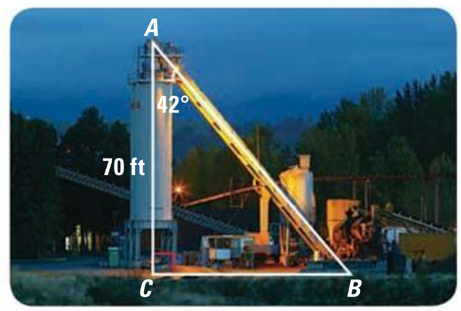
a. $m\angle A = \sin^{-1} 0.87 \approx 60.5°$ **b.** $m\angle B = \cos^{-1} 0.15 \approx 81.4°$

✓ GUIDED PRACTICE for Examples 1 and 2

1. Look back at Example 1. Use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.
2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.54$.

EXAMPLE 3 Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

STEP 1 **Find** $m\angle B$ by using the Triangle Sum Theorem.

$180° = 90° + 42° + m\angle B$

$48° = m\angle B$

STEP 2 **Approximate** BC by using a tangent ratio.

$\tan 42° = \frac{BC}{70}$ Write ratio for tangent of 42°.

$70 \cdot \tan 42° = BC$ Multiply each side by 70.

$70 \cdot 0.9004 \approx BC$ Approximate tan 42°.

$63 \approx BC$ Simplify and round answer.

ANOTHER WAY

You could also find *AB* by using the Pythagorean Theorem, or a sine ratio.

STEP 3 **Approximate** AB using a cosine ratio.

$\cos 42° = \frac{70}{AB}$ Write ratio for cosine of 42°.

$AB \cdot \cos 42° = 70$ Multiply each side by *AB*.

$AB = \frac{70}{\cos 42°}$ Divide each side by cos 42°.

$AB \approx \frac{70}{0.7431}$ Use a calculator to find cos 42°.

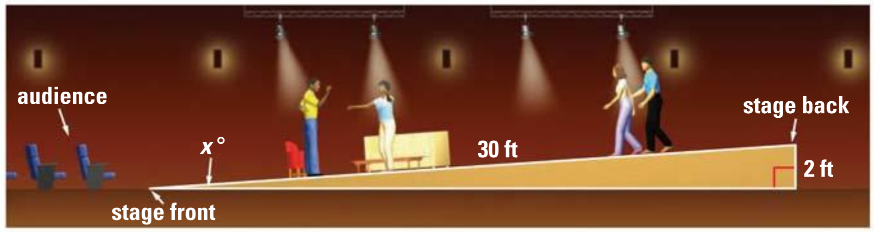
$AB \approx 94.2$ Simplify .

▶ The angle measures are 42°, 48°, and 90°. The side lengths are 70 feet, about 63 feet, and about 94 feet.

EXAMPLE 4 Solve a real-world problem

READ VOCABULARY
A *raked stage* slants upward from front to back to give the audience a better view.

THEATER DESIGN Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?



Solution

Use the sine and inverse sine ratios to find the degree measure x of the rake.

$$\sin x° = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{30} \approx 0.0667$$

$$x \approx \sin^{-1} 0.0667 \approx 3.824$$

▶ The rake is about 3.8°, so it is within the suggested range of 5° or less.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Solve a right triangle that has a 40° angle and a 20 inch hypotenuse.

4. **WHAT IF?** In Example 4, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is this raked stage safe? *Explain*.

7.7 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 29, 30, 35, 40, and 41
◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

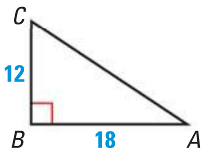
SKILL PRACTICE

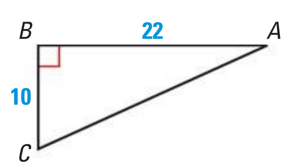
1. **VOCABULARY** Copy and complete: To solve a right triangle means to find the measures of all of its __?__ and __?__.

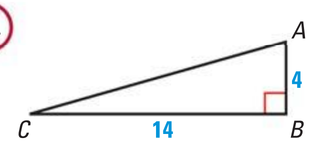
2. ★ **WRITING** *Explain* when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem.

EXAMPLE 1
on p. 483 for Exs. 3–5

USING INVERSE TANGENTS **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

3. 

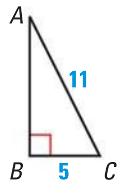
4. 

5. 

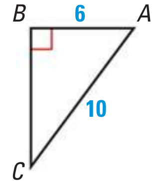
EXAMPLE 2
on p. 484
for Exs. 6–9

USING INVERSE SINES AND COSINES **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

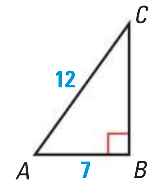
6.



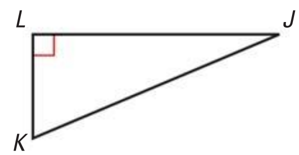
7.



8.



9. ★ **MULTIPLE CHOICE** Which expression is correct?



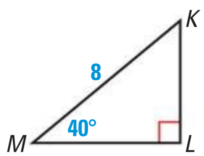
Ⓐ $\sin^{-1}\frac{JL}{JK} = m\angle J$ Ⓑ $\tan^{-1}\frac{KL}{JL} = m\angle J$

Ⓒ $\cos^{-1}\frac{JL}{JK} = m\angle K$ Ⓓ $\sin^{-1}\frac{JL}{KL} = m\angle K$

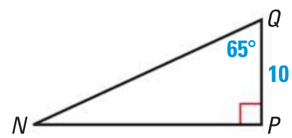
EXAMPLE 3
on p. 484
for Exs. 10–18

SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimal answers to the nearest tenth.**

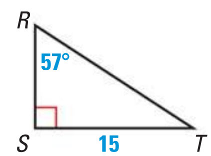
10.



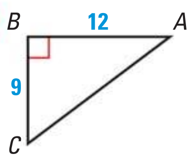
11.



12.



13.



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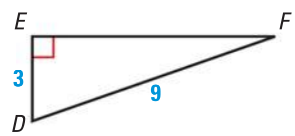

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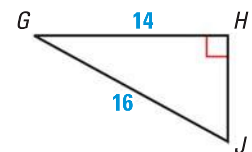
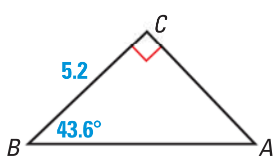
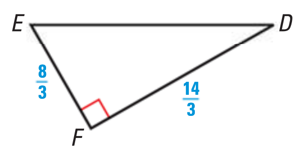


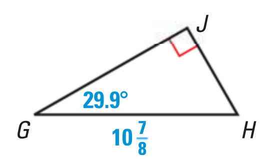
16.



17.

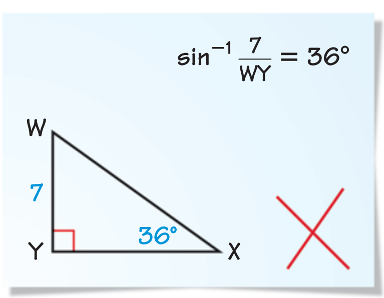


18.

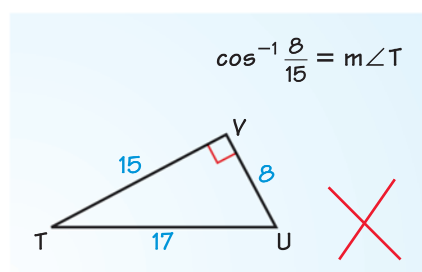


ERROR ANALYSIS ***Describe*** **and correct the student's error in using an inverse trigonometric ratio.**

19.



20.



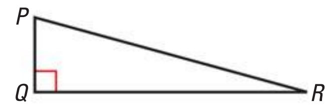
CALCULATOR **Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.**

21. $\sin A = 0.5$ **22.** $\sin A = 0.75$ **23.** $\cos A = 0.33$ **24.** $\cos A = 0.64$

25. $\tan A = 1.0$ **26.** $\tan A = 0.28$ **27.** $\sin A = 0.19$ **28.** $\cos A = 0.81$

◯ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

29. ★ **MULTIPLE CHOICE** Which additional information would *not* be enough to solve $\triangle PRQ$?



Ⓐ $m\angle P$ and PR Ⓑ $m\angle P$ and $m\angle R$

Ⓒ PQ and PR Ⓓ $m\angle P$ and PQ

30. ★ **WRITING** *Explain* why it is incorrect to say that $\tan^{-1} x = \frac{1}{\tan x}$.

31. **SPECIAL RIGHT TRIANGLES** If $\sin A = \frac{1}{2}\sqrt{2}$, what is $m\angle A$? If $\sin B = \frac{1}{2}\sqrt{3}$, what is $m\angle B$?

32. **TRIGONOMETRIC VALUES** Use the *Table of Trigonometric Ratios* on page 925 to answer the questions.

a. What angles have nearly the same sine and tangent values?

b. What angle has the greatest difference in its sine and tangent value?

c. What angle has a tangent value that is double its sine value?

d. Is $\sin 2x$ equal to $2 \cdot \sin x$?

33. **CHALLENGE** The perimeter of rectangle $ABCD$ is 16 centimeters, and the ratio of its width to its length is $1:3$. Segment BD divides the rectangle into two congruent triangles. Find the side lengths and angle measures of one of these triangles.

PROBLEM SOLVING

EXAMPLE 4
on p. 485
for Exs. 34–36

34. **SOCCER** A soccer ball is placed 10 feet away from the goal, which is 8 feet high. You kick the ball and it hits the crossbar along the top of the goal. What is the angle of elevation of your kick?

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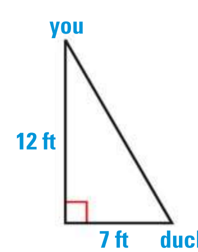
35. ★ **SHORT RESPONSE** You are standing on a footbridge in a city park that is 12 feet high above a pond. You look down and see a duck in the water 7 feet away from the footbridge. What is the angle of depression? *Explain* your reasoning.



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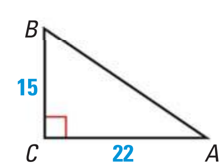
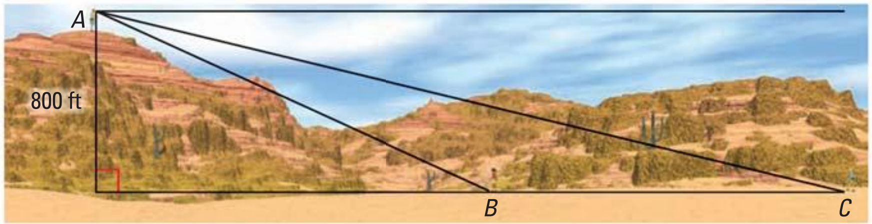
36. **CLAY** In order to unload clay easily, the body of a dump truck must be elevated to at least 55°. If the body of the dump truck is 14 feet long and has been raised 10 feet, will the clay pour out easily?

37. **REASONING** For $\triangle ABC$ shown, each of the expressions $\sin^{-1}\frac{BC}{AB}$, $\cos^{-1}\frac{AC}{AB}$, and $\tan^{-1}\frac{BC}{AC}$ can be used to approximate the measure of $\angle A$. Which expression would you choose? *Explain* your choice.



38. **MULTI-STEP PROBLEM** You are standing on a plateau that is 800 feet above a basin where you can see two hikers.

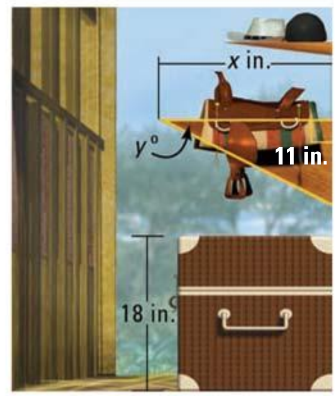


a. If the angle of depression from your line of sight to the hiker at B is 25°, how far is the hiker from the base of the plateau?

b. If the angle of depression from your line of sight to the hiker at C is 15°, how far is the hiker from the base of the plateau?

c. How far apart are the two hikers? *Explain*.

39. **MULTIPLE REPRESENTATIONS** A local ranch offers trail rides to the public. It has a variety of different sized saddles to meet the needs of horse and rider. You are going to build saddle racks that are 11 inches high. To save wood, you decide to make each rack fit each saddle.



a. **Making a Table** The lengths of the saddles range from 20 inches to 27 inches. Make a table showing the saddle rack length x and the measure of the adjacent angle $y°$.

b. **Drawing a Graph** Use your table to draw a scatterplot.

c. **Making a Conjecture** Make a conjecture about the relationship between the length of the rack and the angle needed.

40. ★ **OPEN-ENDED MATH** *Describe* a real-world problem you could solve using a trigonometric ratio.

41. ★ **EXTENDED RESPONSE** Your town is building a wind generator to create electricity for your school. The builder wants your geometry class to make sure that the guy wires are placed so that the tower is secure. By safety guidelines, the distance along the ground from the tower to the guy wire's connection with the ground should be between 50% to 75% of the height of the guy wire's connection with the tower.

a. The tower is 64 feet tall. The builders plan to have the distance along the ground from the tower to the guy wire's connection with the ground be 60% of the height of the tower. How far apart are the tower and the ground connection of the wire?

b. How long will a guy wire need to be that is attached 60 feet above the ground?

c. How long will a guy wire need to be that is attached 30 feet above the ground?

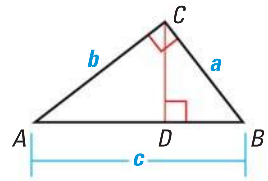
d. Find the angle of elevation of each wire. Are the right triangles formed by the ground, tower, and wires *congruent*, *similar*, or *neither*? *Explain*.

e. *Explain* which trigonometric ratios you used to solve the problem.

42. **CHALLENGE** Use the diagram of $\triangle ABC$.

GIVEN ▶ $\triangle ABC$ with altitude $\overline{CD}$.

PROVE ▶ $\frac{\sin A}{a} = \frac{\sin B}{b}$



MIXED REVIEW

PREVIEW
Prepare for Lesson 8.1 in Ex. 43.

43. Copy and complete the table. *(p. 42)*

Number of sides	Type of polygon
5	?
12	?
?	Octagon
?	Triangle
7	?

Number of sides	Type of polygon
?	*n*-gon
?	Quadrilateral
10	?
9	?
?	Hexagon

A point on an image and the transformation are given. Find the corresponding point on the original figure. *(p. 272)*

44. Point on image: (5, 1); translation: $(x, y) \to (x + 3, y - 2)$

45. Point on image: (4, −6); reflection: $(x, y) \to (x, -y)$

46. Point on image: (−2, 3); translation: $(x, y) \to (x - 5, y + 7)$

Draw a dilation of the polygon with the given vertices using the given scale factor *k*. *(p. 409)*

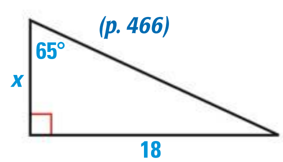
47. $A(2, 2), B(-1, -3), C(5, -3); k = 2$

48. $A(-4, -2), B(-2, 4), C(3, 6), D(6, 3); k = \frac{1}{2}$

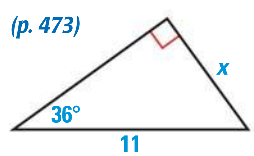
QUIZ for Lessons 7.5–7.7

Find the value of *x* to the nearest tenth.

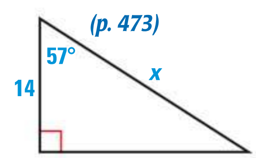
1. *(p. 466)*



2. *(p. 473)*

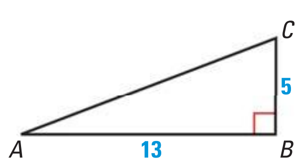


3. *(p. 473)*

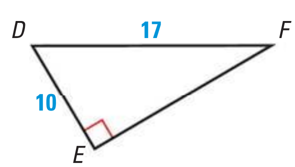


Solve the right triangle. Round decimal answers to the nearest tenth. *(p. 483)*

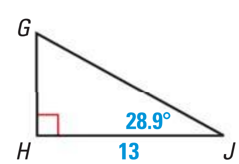
4.



5.



6.



Extension

Use after Lesson 7.7

Law of Sines and Law of Cosines

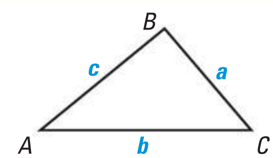
GOAL Use trigonometry with acute and obtuse triangles.

The trigonometric ratios you have seen so far in this chapter can be used to find angle and side measures in right triangles. You can use the Law of Sines to find angle and side measures in *any* triangle.

KEY CONCEPT — *For Your Notebook*

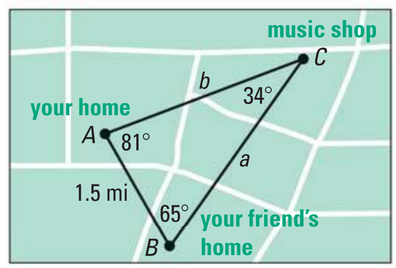
Law of Sines

If $\triangle ABC$ has sides of length a, b, and c as shown, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



EXAMPLE 1 Find a distance using Law of Sines

DISTANCE Use the information in the diagram to determine how much closer you live to the music store than your friend does.



Solution

STEP 1 **Use** the Law of Sines to find the distance a from your friend's home to the music store.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ — Write Law of Sines.

$\frac{\sin 81°}{a} = \frac{\sin 34°}{1.5}$ — Substitute.

$a \approx 2.6$ — Solve for a.

STEP 2 **Use** the Law of Sines to find the distance b from your home to the music store.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ — Write Law of Sines.

$\frac{\sin 65°}{b} = \frac{\sin 34°}{1.5}$ — Substitute.

$b \approx 2.4$ — Solve for b.

STEP 3 **Subtract** the distances.

$a - b \approx 2.6 - 2.4 = 0.2$

▶ You live about 0.2 miles closer to the music store.

LAW OF COSINES You can also use the Law of Cosines to solve any triangle.

KEY CONCEPT — *For Your Notebook*

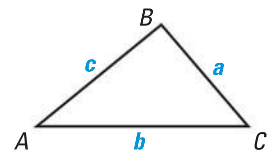
Law of Cosines

If $\triangle ABC$ has sides of length a, b, and c, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

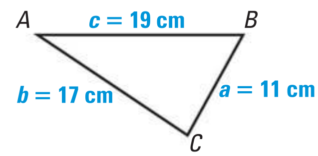
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



EXAMPLE 2 Find an angle measure using Law of Cosines

In $\triangle ABC$ at the right, $a = 11$ cm, $b = 17$ cm, and $c = 19$ cm. Find $m\angle C$.



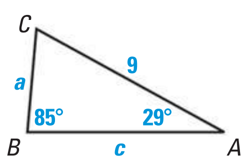
Solution

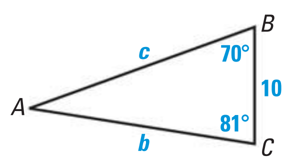
$c^2 = a^2 + b^2 - 2ab \cos C$	**Write Law of Cosines.**
$19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$	**Substitute.**
$0.1310 = \cos C$	**Solve for cos C.**
$m\angle C \approx 82°$	**Find $\cos^{-1}$ (0.1310).**

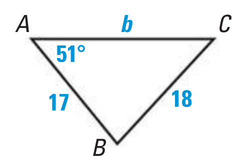
PRACTICE

EXAMPLE 1 for Exs. 1–3

LAW OF SINES **Use the Law of Sines to solve the triangle. Round decimal answers to the nearest tenth.**

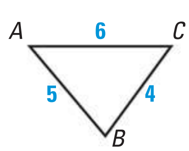
1.


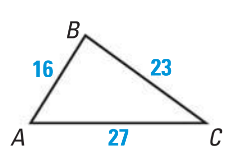
2.


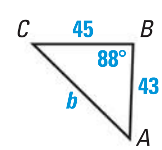
3.


EXAMPLE 2 for Exs. 4–7

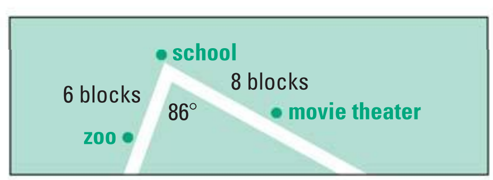
LAW OF COSINES **Use the Law of Cosines to solve the triangle. Round decimal answers to the nearest tenth.**

4.


5.


6.


7. DISTANCE Use the diagram at the right. Find the straight distance between the zoo and movie theater.



MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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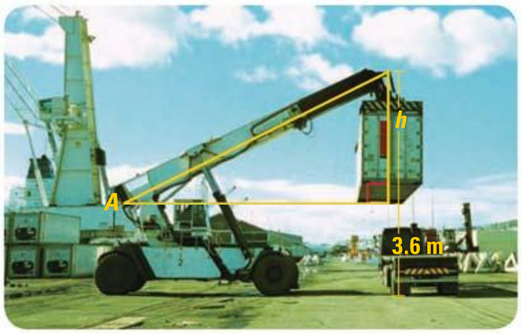
Lessons 7.5–7.7

1. **MULTI-STEP PROBLEM** A *reach stacker* is a vehicle used to lift objects and move them between ships and land.



   a. The vehicle's arm is 10.9 meters long. The maximum measure of $\angle A$ is 60°. What is the greatest height h the arm can reach if the vehicle is 3.6 meters tall?

   b. The vehicle's arm can extend to be 16.4 meters long. What is the greatest height its extended arm can reach?

   c. What is the difference between the two heights the arm can reach above the ground?

2. **EXTENDED RESPONSE** You and a friend are standing the same distance from the edge of a canyon. Your friend looks directly across the canyon at a rock. You stand 10 meters from your friend and estimate the angle between your friend and the rock to be 85°.

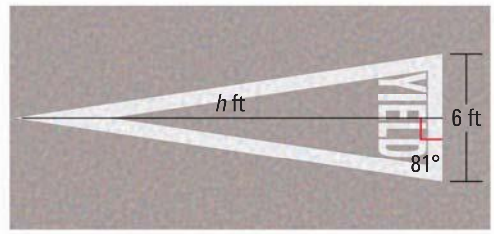
   a. Sketch the situation.

   b. *Explain* how to find the distance across the canyon.

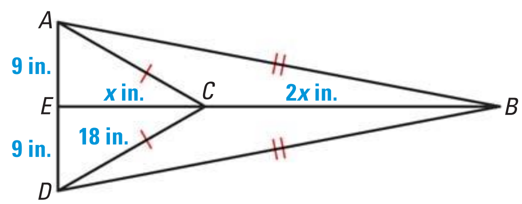
   c. Suppose the actual angle measure is 87°. How far off is your estimate of the distance?

3. **SHORT RESPONSE** The international rules of basketball state the rim of the net should be 3.05 meters above the ground. If your line of sight to the rim is 34° and you are 1.7 meters tall, what is the distance from you to the rim? *Explain* your reasoning.

4. **GRIDDED ANSWER** The specifications for a *yield ahead* pavement marking are shown. Find the height h in feet of this isosceles triangle.



5. **EXTENDED RESPONSE** Use the diagram to answer the questions.

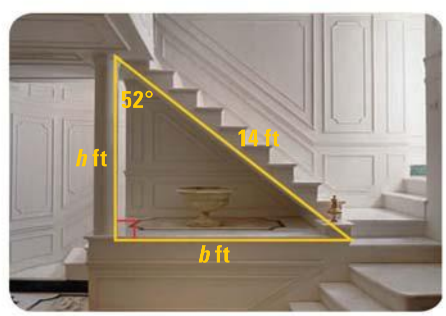


   a. Solve for x. *Explain* the method you chose.

   b. Find $m\angle ABC$. *Explain* the method you chose.

   c. *Explain* a different method for finding each of your answers in parts (a) and (b).

6. **SHORT RESPONSE** The triangle on the staircase below has a 52° angle and the distance along the stairs is 14 feet. What is the height h of the staircase? What is the length b of the base of the staircase?



7. **GRIDDED ANSWER** The base of an isosceles triangle is 70 centimeters long. The altitude to the base is 75 centimeters long. Find the measure of a base angle to the nearest degree.

7 CHAPTER SUMMARY

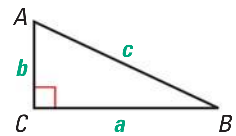
BIG IDEAS — For Your Notebook

Big Idea 1

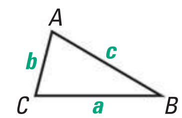
Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b, so that $c^2 = a^2 + b^2$.

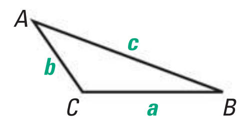
The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.



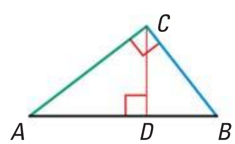
If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Big Idea 2

Using Special Relationships in Right Triangles

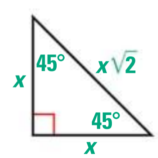
GEOMETRIC MEAN In right $\triangle ABC$, altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Also, $\frac{BD}{CD} = \frac{CD}{AD}$, $\frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.



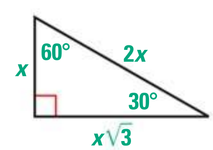
SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle



hypotenuse = leg • $\sqrt{2}$

30°-60°-90° Triangle



hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

Big Idea 3

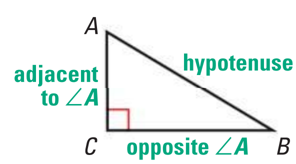
Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of tan $x°$, sin $x°$, and cos $x°$ depend only on the angle measure and not on the side length.

$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{BC}{AC}$ $\qquad \tan^{-1}\frac{BC}{AC} = m\angle A$

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{AB}$ $\qquad \sin^{-1}\frac{BC}{AB} = m\angle A$

$\cos A = \frac{\text{adj.}}{\text{hyp.}} = \frac{AC}{AB}$ $\qquad \cos^{-1}\frac{AC}{AB} = m\angle A$



7 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation __?__.

2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle?

3. **WRITING** *Describe* the difference between an angle of depression and an angle of elevation.

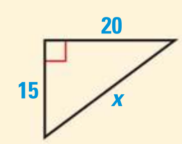
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

7.1 Apply the Pythagorean Theorem *pp. 433–439*

EXAMPLE

Find the value of x.



Because x is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ **Pythagorean Theorem**

$x^2 = 15^2 + 20^2$ **Substitute.**

$x^2 = 625$ **Simplify.**

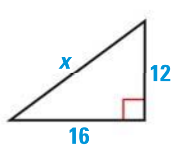
$x = 25$ **Find the positive square root.**

EXERCISES

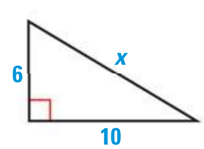
EXAMPLES 1 and 2 on pp. 433–434 for Exs. 4–6

Find the unknown side length x.

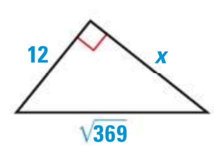
4.



5.



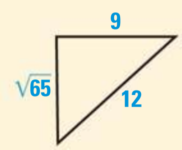
6.



7.2 Use the Converse of the Pythagorean Theorem *pp. 441–447*

EXAMPLE

Tell whether the given triangle is a right triangle.



Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$$12^2 \stackrel{?}{=} (\sqrt{65})^2 + 9^2$$

$$144 \stackrel{?}{=} 65 + 81$$

$$144 < 146$$

The triangle is not a right triangle. It is an acute triangle.

EXERCISES

EXAMPLE 2
on p. 442
for Exs. 7–12

Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*.

7. 6, 8, 9

8. 4, 2, 5

9. $10, 2\sqrt{2}, 6\sqrt{3}$

10. 15, 20, 15

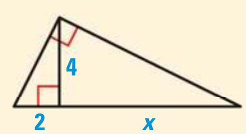
11. $3, 3, 3\sqrt{2}$

12. $13, 18, 3\sqrt{55}$

7.3 Use Similar Right Triangles *pp. 449–456*

EXAMPLE

Find the value of *x*.



By Theorem 7.6, you know that 4 is the geometric mean of x and 2.

$\frac{x}{4} = \frac{4}{2}$ **Write a proportion.**

$2x = 16$ **Cross Products Property**

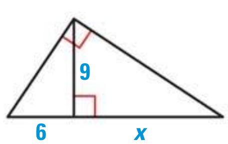
$x = 8$ **Divide.**

EXERCISES

EXAMPLES 2 and 3
on pp. 450–451
for Exs. 13–18

Find the value of *x*.

13.



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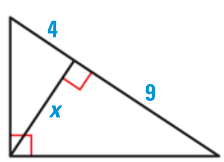

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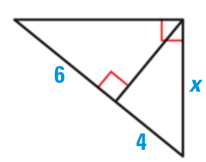

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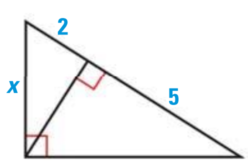
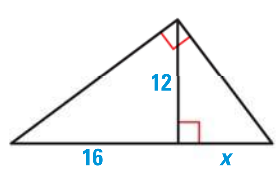
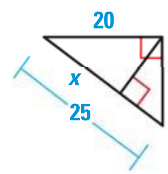


17.



18.



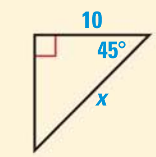
7 CHAPTER REVIEW

7.4 Special Right Triangles

pp. 457–464

EXAMPLE

Find the length of the hypotenuse.



By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle.

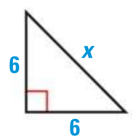
hypotenuse $=$ leg $\cdot \sqrt{2}$	**45°-45°-90° Triangle Theorem**
$x = 10\sqrt{2}$	**Substitute.**

EXERCISES

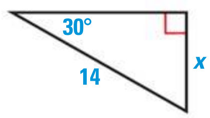
EXAMPLES 1, 2, and 5
on pp. 457–459
for Exs. 19–21

Find the value of *x*. Write your answer in simplest radical form.

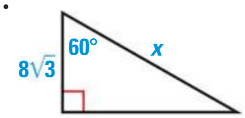
19.



20.



21.



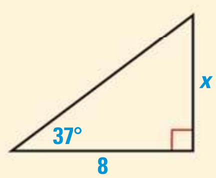
7.5 Apply the Tangent Ratio

pp. 466–472

EXAMPLE

Find the value of *x*.

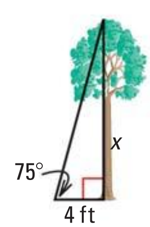
$\tan 37° = \frac{\text{opp.}}{\text{adj.}}$	**Write ratio for tangent of 37°.**
$\tan 37° = \frac{x}{8}$	**Substitute.**
$8 \cdot \tan 37° = x$	**Multiply each side by 8.**
$6 \approx x$	**Use a calculator to simplify.**



EXERCISES

EXAMPLE 2
on p. 467
for Exs. 22–26

In Exercises 22 and 23, use the diagram.

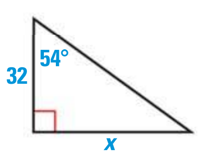


22. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.

23. In Exercise 22, how tall is the tree if the angle is 55°?

Find the value of *x* to the nearest tenth.

24.



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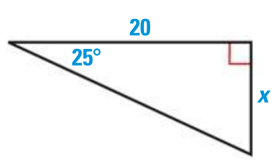
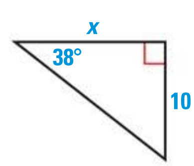


26.



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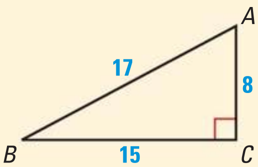
Chapter Review Practice

7.6 Apply the Sine and Cosine Ratios

pp. 473–480

EXAMPLE

Find sin A and sin B.



$$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824$$

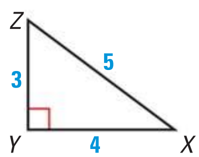
$$\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706$$

EXERCISES

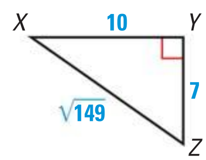
EXAMPLES 1 and 2 on pp. 473–474 for Exs. 27–29

Find sin X and cos X. Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

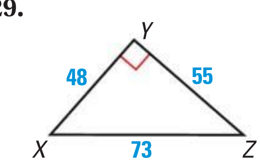
27.



28.



29.

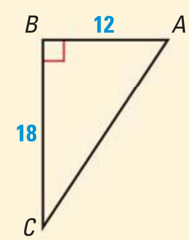


7.7 Solve Right Triangles

pp. 483–489

EXAMPLE

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A = \frac{18}{12} = \frac{3}{2} = 1.5$, $\tan^{-1} 1.5 = m\angle A$.

Use a calculator to evaluate this expression.

$\tan^{-1} 1.5 \approx 56.3099324\ldots$

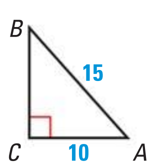
So, the measure of $\angle A$ is approximately 56.3°.

EXERCISES

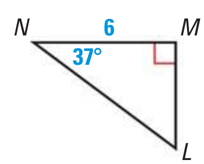
EXAMPLE 3 on p. 484 for Exs. 30–33

Solve the right triangle. Round decimal answers to the nearest tenth.

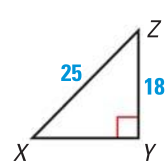
30.



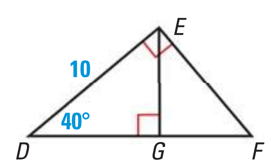
31.



32.



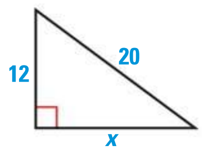
33. Find the measures of $\angle GED$, $\angle GEF$, and $\angle EFG$. Find the lengths of $\overline{EG}$, $\overline{DF}$, $\overline{EF}$.



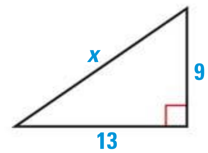
7 CHAPTER TEST

Find the value of *x*. Write your answer in simplest radical form.

1.



2.



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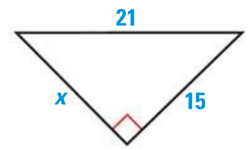


Classify the triangle as *acute*, *right*, or *obtuse*.

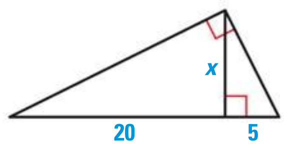
4. 5, 15, $5\sqrt{10}$

5. 4.3, 6.7, 8.2

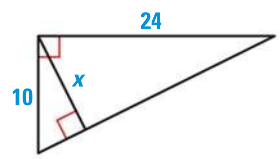
6. 5, 7, 8

Find the value of *x*. Round decimal answers to the nearest tenth.

7.



8.



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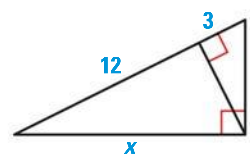
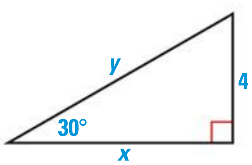


Find the value of each variable. Write your answer in simplest radical form.

10.



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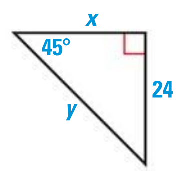

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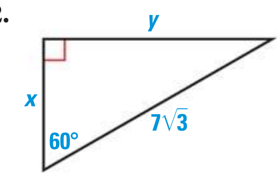
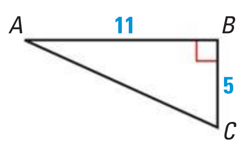


Solve the right triangle. Round decimal answers to the nearest tenth.

13.



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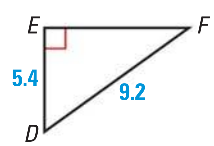
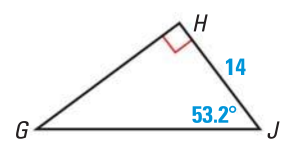
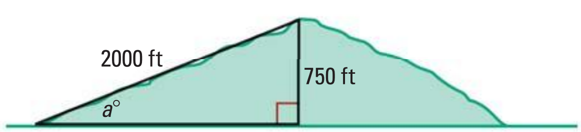


15.



16. **FLAGPOLE** Julie is 6 feet tall. If she stands 15 feet from the flagpole and holds a cardboard square, the edges of the square line up with the top and bottom of the flagpole. Approximate the height of the flagpole.

17. **HILLS** The length of a hill in your neighborhood is 2000 feet. The height of the hill is 750 feet. What is the angle of elevation of the hill?



7 ALGEBRA REVIEW

Animated Algebra classzone.com

GRAPH AND SOLVE QUADRATIC EQUATIONS

The graph of $y = ax^2 + bx + c$ is a parabola that opens upward if $a > 0$ and opens downward if $a < 0$. The x-coordinate of the vertex is $-\frac{b}{2a}$. The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

EXAMPLE 1 Graph a quadratic function

Graph the equation $y = -x^2 + 4x - 3$.

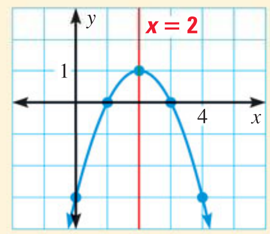
Because $a = -1$ and $-1 < 0$, the graph opens downward.

The vertex has x-coordinate $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$.

The y-coordinate of the vertex is $-(2)^2 + 4(2) - 3 = 1$.

So, the vertex is (2, 1) and the axis of symmetry is $x = 2$.

Use a table of values to draw a parabola through the plotted points.



EXAMPLE 2 Solve a quadratic equation by graphing

Solve the equation $x^2 - 2x = 3$.

Write the equation in the standard form $ax^2 + bx + c = 0$:

$x^2 - 2x - 3 = 0$.

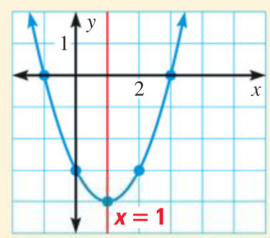
Graph the related quadratic function $y = x^2 - 2x - 3$, as shown.

The x-intercepts of the graph are -1 and 3.

So, the solutions of $x^2 - 2x = 3$ are -1 and 3.

Check the solution algebraically.

$(-1)^2 - 2(-1) \stackrel{?}{=} 3 \rightarrow 1 + 2 = 3$ $\qquad$ $(3)^2 - 2(3) \stackrel{?}{=} 3 \rightarrow 9 - 6 = 3$ ✓



EXERCISES

EXAMPLE 1 for Exs. 1–6

Graph the quadratic function. Label the vertex and axis of symmetry.

1. $y = x^2 - 6x + 8$
2. $y = -x^2 - 4x + 2$
3. $y = 2x^2 - x - 1$
4. $y = 3x^2 - 9x + 2$
5. $y = \frac{1}{2}x^2 - x + 3$
6. $y = -4x^2 + 6x - 5$

EXAMPLE 2 for Exs. 7–18

Solve the quadratic equation by graphing. Check solutions algebraically.

7. $x^2 = x + 6$
8. $4x + 4 = -x^2$
9. $2x^2 = -8$
10. $3x^2 + 2 = 14$
11. $-x^2 + 4x - 5 = 0$
12. $2x - x^2 = -15$
13. $\frac{1}{4}x^2 = 2x$
14. $x^2 + 3x = 4$
15. $x^2 + 8 = 6x$
16. $x^2 = 9x - 1$
17. $-25 = x^2 + 10x$
18. $x^2 + 6x = 0$

7 ★ Standardized TEST PREPARATION

MULTIPLE CHOICE QUESTIONS

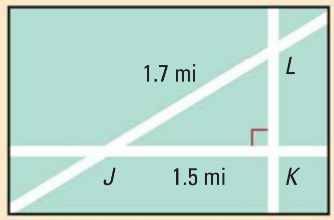
If you have difficulty solving a multiple choice question directly, you may be able to use another approach to eliminate incorrect answer choices and obtain the correct answer.

PROBLEM 1

You ride your bike at an average speed of 10 miles per hour. How long does it take you to ride one time around the triangular park shown in the diagram?

Ⓐ 0.1 h Ⓑ 0.2 h

Ⓒ 0.3 h Ⓓ 0.4 h



METHOD 1

SOLVE DIRECTLY The park is a right triangle. Use the Pythagorean Theorem to find KL. Find the perimeter of $\triangle JKL$. Then find how long it takes to ride around the park.

STEP 1 **Find** KL. Use the Pythagorean Theorem.

$$JK^2 + KL^2 = JL^2$$
$$1.5^2 + KL^2 = 1.7^2$$
$$2.25 + KL^2 = 2.89$$
$$KL^2 = 0.64$$
$$KL = 0.8$$

STEP 2 **Find** the perimeter of $\triangle JKL$.

$$P = JK + JL + KL$$
$$= 1.5 + 1.7 + 0.8$$
$$= 4 \text{ mi}$$

STEP 3 **Find** the time t (in hours) it takes you to go around the park.

$$\text{Rate} \times \text{Time} = \text{Distance}$$
$$(10 \text{ mi/h}) \cdot t = 4 \text{ mi}$$
$$t = 0.4 \text{ h}$$

The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Another method is to find how far you can travel in the given times to eliminate choices that are not reasonable.

STEP 1 **Find** how far you will travel in each of the given times. Use the formula $rt = d$.

Choice A: $0.1(10) = 1$ mi

Choice B: $0.2(10) = 2$ mi

Choice C: $0.3(10) = 3$ mi

Choice D: $0.4(10) = 4$ mi

The distance around two sides of the park is $1.5 + 1.7 = 3.2$ mi. But you need to travel around all three sides, which is longer.

Since $1 < 3.2$, $2 < 3.2$, and $3 < 3.2$. You can eliminate choices A, B, and C.

STEP 2 **Check** that D is the correct answer. If the distance around the park is 4 miles, then

$$KL = 4 - JK - JL$$
$$= 4 - 1.5 - 1.7 = 0.8 \text{ mi.}$$

Apply the Converse of the Pythagorean Theorem.

$$0.8^2 + 1.5^2 \stackrel{?}{=} 1.7^2$$
$$0.64 + 2.25 \stackrel{?}{=} 2.89$$
$$2.89 = 2.89 \checkmark$$

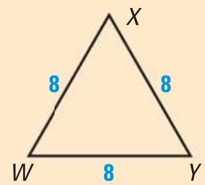
The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

PROBLEM 2

What is the height of $\triangle WXY$?

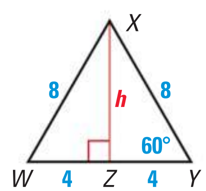
Ⓐ 4 Ⓑ $4\sqrt{3}$

Ⓒ 8 Ⓓ $8\sqrt{3}$



METHOD 1

SOLVE DIRECTLY Draw altitude $\overline{XZ}$ to form two congruent 30°-60°-90° triangles.



Let h be the length of the longer leg of $\triangle XZY$. The length of the shorter leg is 4.

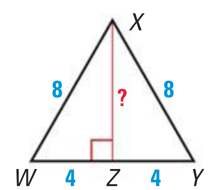
$$\text{longer leg} = \sqrt{3} \cdot \text{shorter leg}$$

$$h = 4\sqrt{3}$$

The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Another method is to use theorems about triangles to eliminate incorrect choices. Draw altitude $\overline{XZ}$ to form two congruent right triangles.



Consider $\triangle XZW$. By the Triangle Inequality, $XW < WZ + XZ$. So, $8 < 4 + XZ$ and $XZ > 4$. You can eliminate choice A. Also, XZ must be less than the hypotenuse of $\triangle XWZ$. You can eliminate choices C and D.

The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

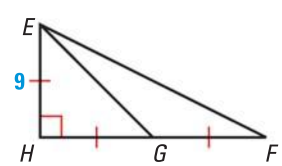
PRACTICE

Explain **why you can eliminate the highlighted answer choice.**

1. In the figure shown, what is the length of $\overline{EF}$?

Ⓐ 9 Ⓑ ✗ $9\sqrt{2}$

Ⓒ 18 Ⓓ $9\sqrt{5}$



2. Which of the following lengths are side lengths of a right triangle?

Ⓐ ✗ 2, 21, 23 Ⓑ 3, 4, 5 Ⓒ 9, 16, 18 Ⓓ 11, 16, 61

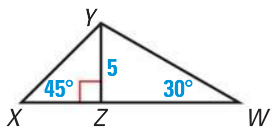
3. In $\triangle PQR$, $PQ = QR = 13$ and $PR = 10$. What is the length of the altitude drawn from vertex Q?

Ⓐ 10 Ⓑ 11 Ⓒ 12 Ⓓ ✗

7 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

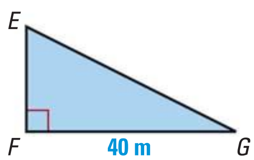
1. Which expression gives the correct length for XW in the diagram below?



Ⓐ $5 + 5\sqrt{2}$ Ⓑ $5 + 5\sqrt{3}$

Ⓒ $5\sqrt{3} + 5\sqrt{2}$ Ⓓ $5 + 10$

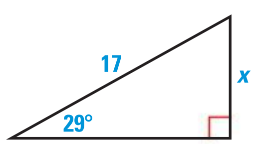
2. The area of $\triangle EFG$ is 400 square meters. To the nearest tenth of a meter, what is the length of side $\overline{EG}$?



Ⓐ 10.0 meters Ⓑ 20.0 meters

Ⓒ 44.7 meters Ⓓ 56.7 meters

3. Which expression can be used to find the value of x in the diagram below?



Ⓐ $\tan 29° = \frac{x}{17}$ Ⓑ $\cos 29° = \frac{x}{17}$

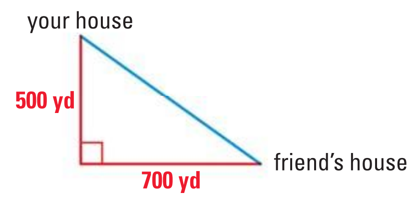
Ⓒ $\tan 61° = \frac{x}{17}$ Ⓓ $\cos 61° = \frac{x}{17}$

4. A fire station, a police station, and a hospital are not positioned in a straight line. The distance from the police station to the fire station is 4 miles. The distance from the fire station to the hospital is 3 miles. Which of the following could *not* be the distance from the police station to the hospital?

Ⓐ 1 mile Ⓑ 2 miles

Ⓒ 5 miles Ⓓ 6 miles

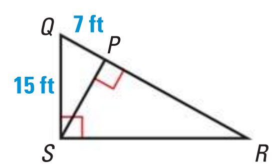
5. It takes 14 minutes to walk from your house to your friend's house on the path shown in red. If you walk at the same speed, about how many minutes will it take on the path shown in blue?



Ⓐ 6 minutes Ⓑ 8 minutes

Ⓒ 10 minutes Ⓓ 13 minutes

6. Which equation can be used to find QR in the diagram below?



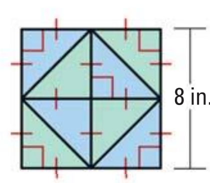
Ⓐ $\frac{QR}{15} = \frac{15}{7}$

Ⓑ $\frac{15}{QR} = \frac{QR}{8}$

Ⓒ $QR = \sqrt{15^2 + 27^2}$

Ⓓ $\frac{QR}{7} = \frac{7}{15}$

7. Stitches are sewn along the black line segments in the potholder shown below. There are 10 stitches per inch. Which is the closest estimate of the number of stitches used?

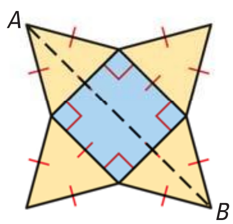


Ⓐ 480 Ⓑ 550

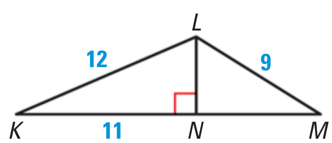
Ⓒ 656 Ⓓ 700

GRIDDED ANSWER

8. A design on a T-shirt is made of a square and four equilateral triangles. The side length of the square is 4 inches. Find the distance (in inches) from point A to point B. Round to the nearest tenth.

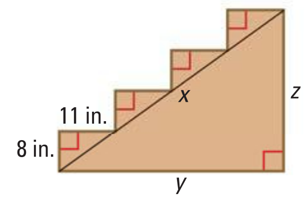


9. Use the diagram below. Find KM to the nearest tenth of a unit.

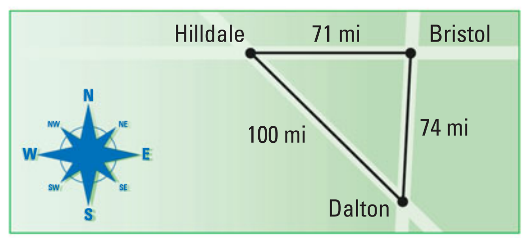


SHORT RESPONSE

10. The diagram shows the side of a set of stairs. In the diagram, the smaller right triangles are congruent. *Explain* how to find the lengths x, y, and z.



11. You drive due north from Dalton to Bristol. Next, you drive from Bristol to Hilldale. Finally, you drive from Hilldale to Dalton. Is Hilldale due west of Bristol? *Explain*.

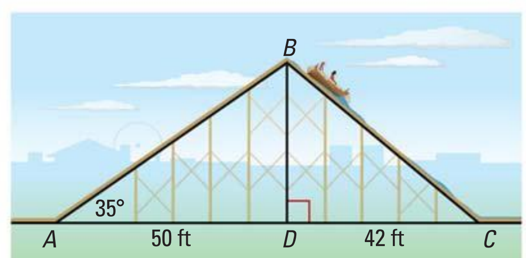


EXTENDED RESPONSE

12. The design for part of a water ride at an amusement park is shown. The ride carries people up a track along ramp $\overline{AB}$. Then riders travel down a water chute along ramp $\overline{BC}$.

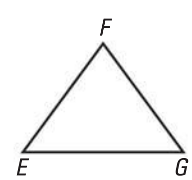
a. How high is the ride above point D? *Explain*.

b. What is the total distance from point A to point B to point C? *Explain*.



13. A formula for the area A of a triangle is *Heron's Formula*. For a triangle with side lengths EF, FG, and EG, the formula is

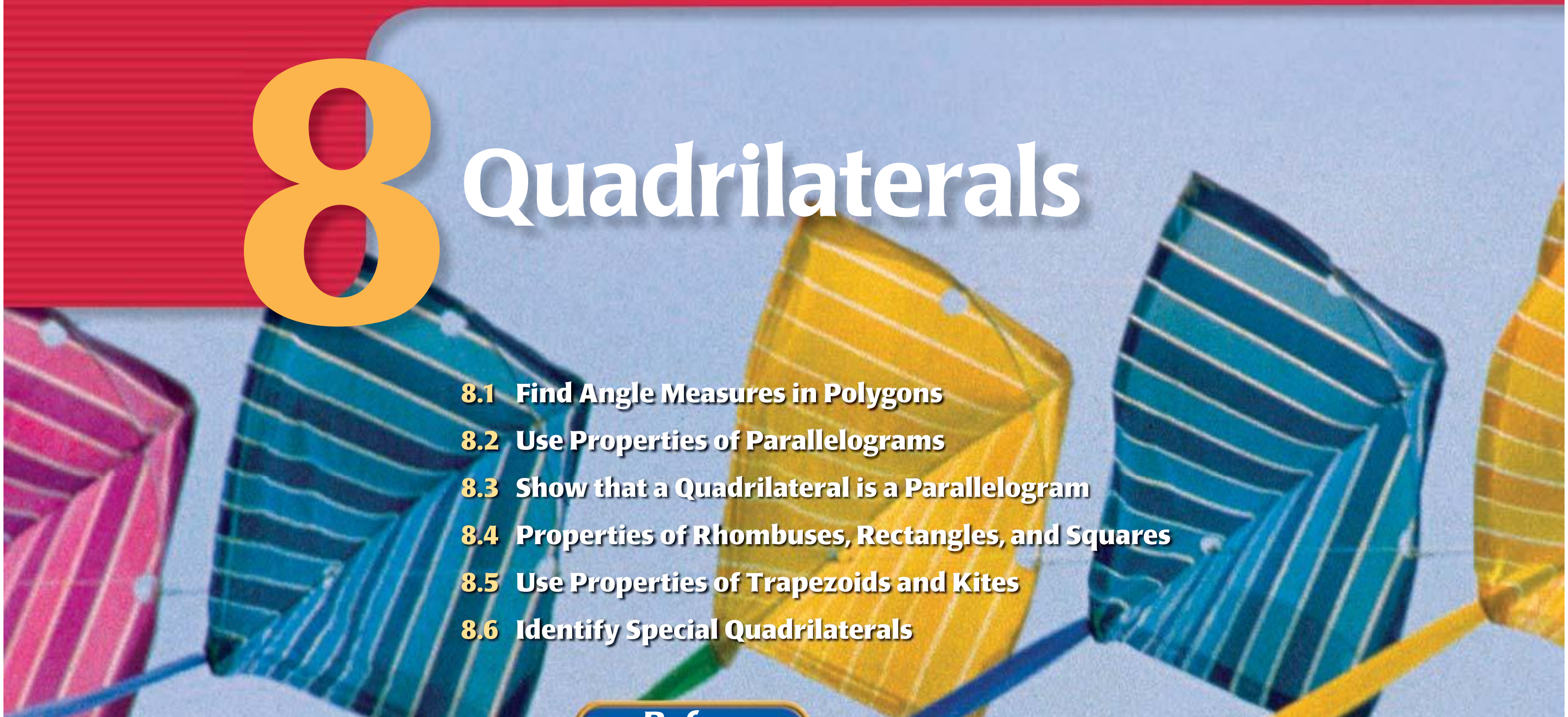
$$A = \sqrt{s(s - EF)(s - FG)(s - EG)}, \text{ where } s = \frac{1}{2}(EF + FG + EG).$$



a. In $\triangle EFG$ shown, $EF = FG = 15$, and $EG = 18$. Use Heron's formula to find the area of $\triangle EFG$. Round to the nearest tenth.

b. Use the formula $A = \frac{1}{2}bh$ to find the area of $\triangle EFG$. Round to the nearest tenth.

c. Use Heron's formula to *justify* that the area of an equilateral triangle with side length x is $A = \frac{x^2}{4}\sqrt{3}$.



8 Quadrilaterals

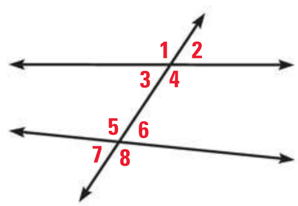
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 8: identifying angle pairs, using the Triangle Sum Theorem, and using parallel lines.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

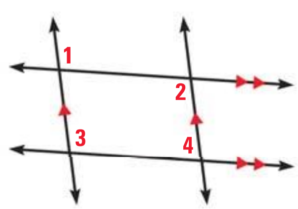


1. $\angle 1$ and __?__ are vertical angles.
2. $\angle 3$ and __?__ are consecutive interior angles.
3. $\angle 7$ and __?__ are corresponding angles.
4. $\angle 5$ and __?__ are alternate interior angles.

SKILLS AND ALGEBRA CHECK

5. In $\triangle ABC$, $m\angle A = x°$, $m\angle B = 3x°$, and $m\angle C = (4x - 12)°$. Find the measures of the three angles. ***(Review p. 217 for 8.1.)***

Find the measure of the indicated angle. ***(Review p. 154 for 8.2–8.5.)***



6. If $m\angle 3 = 105°$, then $m\angle 2 =$ __?__.
7. If $m\angle 1 = 98°$, then $m\angle 3 =$ __?__.
8. If $m\angle 4 = 82°$, then $m\angle 1 =$ __?__.
9. If $m\angle 2 = 102°$, then $m\angle 4 =$ __?__.

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 8, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 559. You will also use the key vocabulary listed below.

Big Ideas

1. Using angle relationships in polygons
2. Using properties of parallelograms
3. Classifying quadrilaterals by their properties

KEY VOCABULARY

- diagonal, *p. 507*
- parallelogram, *p. 515*
- rhombus, *p. 533*
- rectangle, *p. 533*
- square, *p. 533*
- trapezoid, *p. 542*
  bases, base angles, legs
- isosceles trapezoid, *p. 543*
- midsegment of a trapezoid, *p. 544*
- kite, *p. 545*

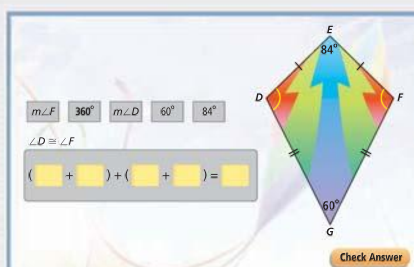
Why?

You can use properties of quadrilaterals and other polygons to find side lengths and angle measures.

Animated Geometry

The animation illustrated below for Example 4 on page 545 helps you answer this question: How can classifying a quadrilateral help you draw conclusions about its sides and angles?

Many real-world kites are shaped like geometric kites.



Use properties of quadrilaterals to write an equation about the angle measures.

Animated Geometry at classzone.com

Other animations for Chapter 8: pages 509, 519, 527, 535, 551, and 553

Investigating Geometry **ACTIVITY** *Use before Lesson 8.1*

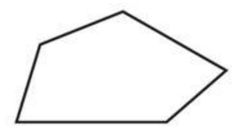
8.1 Investigate Angle Sums in Polygons

MATERIALS • straightedge • ruler

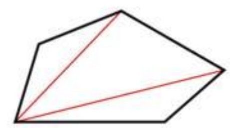
QUESTION **What is the sum of the measures of the interior angles of a convex n-gon?**

Recall from page 43 that an n-gon is a polygon with n sides and n vertices.

EXPLORE **Find sums of interior angle measures**



STEP 1 ***Draw polygons*** Use a straightedge to draw convex polygons with three sides, four sides, five sides, and six sides. An example is shown.



STEP 2 ***Draw diagonals*** In each polygon, draw all the diagonals from one vertex. A *diagonal* is a segment that joins two nonconsecutive vertices. Notice that the diagonals divide the polygon into triangles.

STEP 3 ***Make a table*** Copy the table below. By the Triangle Sum Theorem, the sum of the measures of the interior angles of a triangle is 180°. Use this theorem to complete the table.

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Triangle	3	1	$1 \cdot 180° = 180°$
Quadrilateral	?	?	$2 \cdot 180° = 360°$
Pentagon	?	?	?
Hexagon	?	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

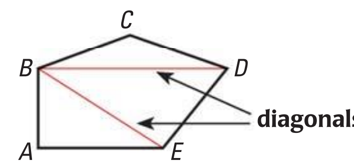
1. Look for a pattern in the last column of the table. What is the sum of the measures of the interior angles of a convex heptagon? a convex octagon? *Explain* your reasoning.
2. Write an expression for the sum of the measures of the interior angles of a convex n-gon.
3. Measure the side lengths in the hexagon you drew. Compare the lengths with those in hexagons drawn by other students. Do the side lengths affect the sum of the interior angle measures of a hexagon? *Explain*.

8.1 Find Angle Measures in Polygons

Before	You classified polygons.
Now	You will find angle measures in polygons.
Why?	So you can describe a baseball park, as in Exs. 28–29.

Key Vocabulary
- **diagonal**
- **interior angle,** *p. 218*
- **exterior angle,** *p. 218*

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two *nonconsecutive vertices*. Polygon $ABCDE$ has two diagonals from vertex B, $\overline{BD}$ and $\overline{BE}$.



As you can see, the diagonals from one vertex form triangles. In the Activity on page 506, you used these triangles to find the sum of the interior angle measures of a polygon. Your results support the following theorem and corollary.

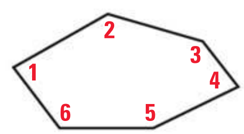
THEOREMS — *For Your Notebook*

THEOREM 8.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180°$

Proof: Ex. 33, p. 512 (for pentagons)



***n* = 6**

COROLLARY TO THEOREM 8.1 Interior Angles of a Quadrilateral

The sum of the measures of the interior angles of a quadrilateral is 360°.

Proof: Ex. 34, p. 512

EXAMPLE 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex octagon.

Solution

An octagon has 8 sides. Use the Polygon Interior Angles Theorem.

$(n - 2) \cdot 180° = (8 - 2) \cdot 180°$ **Substitute 8 for *n*.**

$= 6 \cdot 180°$ **Subtract.**

$= 1080°$ **Multiply.**

▶ The sum of the measures of the interior angles of an octagon is 1080°.

EXAMPLE 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 900°. Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n. Then solve the equation to find the number of sides.

$(n - 2) \cdot 180° = 900°$ Polygon Interior Angles Theorem

$n - 2 = 5$ Divide each side by 180°.

$n = 7$ Add 2 to each side.

▶ The polygon has 7 sides. It is a heptagon.

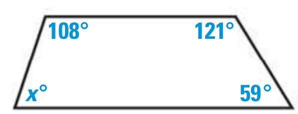
✓ GUIDED PRACTICE for Examples 1 and 2

1. The coin shown is in the shape of a regular 11-gon. Find the sum of the measures of the interior angles.

2. The sum of the measures of the interior angles of a convex polygon is 1440°. Classify the polygon by the number of sides.

EXAMPLE 3 Find an unknown interior angle measure

xy ALGEBRA **Find the value of x in the diagram shown.**



Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x. Then solve the equation.

$x° + 108° + 121° + 59° = 360°$ Corollary to Theorem 8.1

$x + 288 = 360$ Combine like terms.

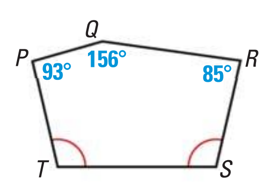
$x = 72$ Subtract 288 from each side.

▶ The value of x is 72.

✓ GUIDED PRACTICE for Example 3

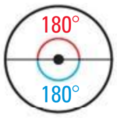
3. Use the diagram at the right. Find $m\angle S$ and $m\angle T$.

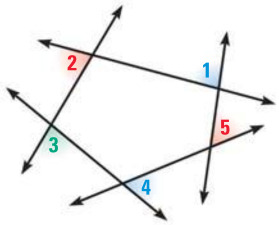
4. The measures of three of the interior angles of a quadrilateral are 89°, 110°, and 46°. Find the measure of the fourth interior angle.



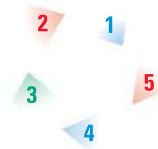
EXTERIOR ANGLES Unlike the sum of the interior angle measures of a convex polygon, the sum of the exterior angle measures does *not* depend on the number of sides of the polygon. The diagrams below suggest that the sum of the measures of the exterior angles, one at each vertex, of a pentagon is 360°. In general, this sum is 360° for any convex polygon.

VISUALIZE IT
A circle contains two straight angles. So, there are 180° + 180°, or 360°, in a circle.

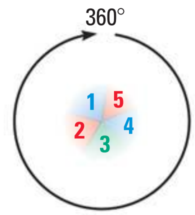




STEP 1 **Shade** one exterior angle at each vertex.



STEP 2 **Cut** out the exterior angles.



STEP 3 **Arrange** the exterior angles to form 360°.

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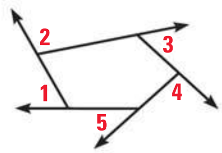
THEOREM *For Your Notebook*

THEOREM 8.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

Proof: Ex. 35, p. 512

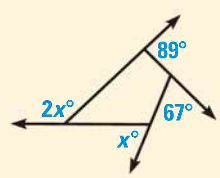


EXAMPLE 4 Standardized Test Practice

What is the value of *x* in the diagram shown?

Ⓐ 67 Ⓑ 68

Ⓒ 91 Ⓓ 136



ELIMINATE CHOICES
You can quickly eliminate choice *D*. If *x* were equal to 136, then the sum of only two of the angle measures ($x°$ and $2x°$) would be greater than 360°.

Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$x° + 2x° + 89° + 67° = 360°$ **Polygon Exterior Angles Theorem**

$3x + 156 = 360$ **Combine like terms.**

$x = 68$ **Solve for *x*.**

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

5. A convex hexagon has exterior angles with measures 34°, 49°, 58°, 67°, and 75°. What is the measure of an exterior angle at the sixth vertex?

EXAMPLE 5 Find angle measures in regular polygons

READ VOCABULARY
Recall that a *dodecagon* is a polygon with 12 sides and 12 vertices.

TRAMPOLINE The trampoline shown is shaped like a regular dodecagon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

Solution

a. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$(n - 2) \cdot 180° = (12 - 2) \cdot 180° = 1800°$

Then find the measure of one interior angle. A regular dodecagon has 12 congruent interior angles. Divide 1800° by 12: $1800° \div 12 = 150°$.

▶ The measure of each interior angle in the dodecagon is 150°.

b. By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is 360°. Divide 360° by 12 to find the measure of one of the 12 congruent exterior angles: $360° \div 12 = 30°$.

▶ The measure of each exterior angle in the dodecagon is 30°.

✓ GUIDED PRACTICE for Example 5

6. An interior angle and an adjacent exterior angle of a polygon form a linear pair. How can you use this fact as another method to find the exterior angle measure in Example 5?

8.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 29
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 23, and 37
◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

SKILL PRACTICE

1. VOCABULARY Sketch a convex hexagon. Draw all of its diagonals.

2. ★ **WRITING** How many exterior angles are there in an n-gon? Are all the exterior angles considered when you use the Polygon Exterior Angles Theorem? *Explain.*

EXAMPLES 1 and 2 on pp. 507–508 for Exs. 3–10

INTERIOR ANGLE SUMS **Find the sum of the measures of the interior angles of the indicated convex polygon.**

3. Nonagon **4.** 14-gon **5.** 16-gon **6.** 20-gon

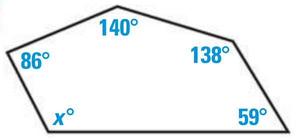
FINDING NUMBER OF SIDES **The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.**

7. 360° **8.** 720° **9.** 1980° **10.** 2340°

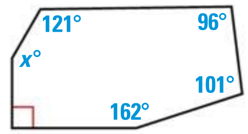
EXAMPLES 3 and 4
on pp. 508–509
for Exs. 11–18

ALGEBRA **Find the value of x.**

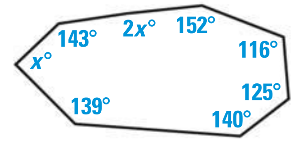
11.



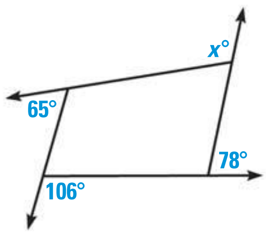
12.



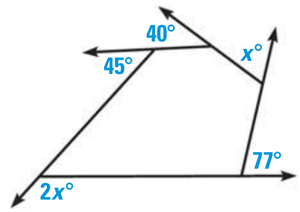
13.



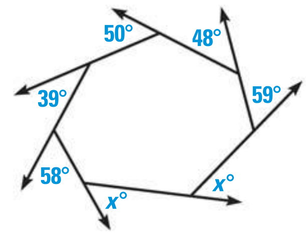
14.



15.



16.



17. **ERROR ANALYSIS** A student claims that the sum of the measures of the exterior angles of an octagon is greater than the sum of the measures of the exterior angles of a hexagon. The student justifies this claim by saying that an octagon has two more sides than a hexagon. *Describe* and correct the error the student is making.

18. ★ **MULTIPLE CHOICE** The measures of the interior angles of a quadrilateral are $x°$, $2x°$, $3x°$, and $4x°$. What is the measure of the largest interior angle?

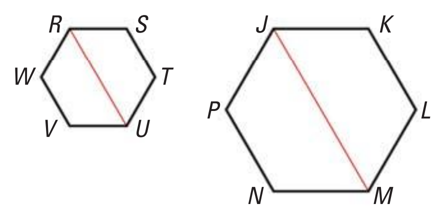
Ⓐ 120° Ⓑ 144° Ⓒ 160° Ⓓ 360°

EXAMPLE 5
on p. 510
for Exs. 19–21

REGULAR POLYGONS **Find the measures of an interior angle and an exterior angle of the indicated regular polygon.**

19. Regular pentagon **20.** Regular 18-gon **21.** Regular 90-gon

22. **DIAGONALS OF SIMILAR FIGURES** Hexagons *RSTUVW* and *JKLMNP* are similar. $\overline{RU}$ and $\overline{JM}$ are diagonals. Given $ST = 6$, $KL = 10$, and $RU = 12$, find JM.



23. ★ **SHORT RESPONSE** *Explain* why any two regular pentagons are similar.

REGULAR POLYGONS **Find the value of n for each regular n-gon described.**

24. Each interior angle of the regular n-gon has a measure of 156°.

25. Each exterior angle of the regular n-gon has a measure of 9°.

26. **POSSIBLE POLYGONS** Determine if it is possible for a regular polygon to have an interior angle with the given angle measure. *Explain* your reasoning.

a. 165° **b.** 171° **c.** 75° **d.** 40°

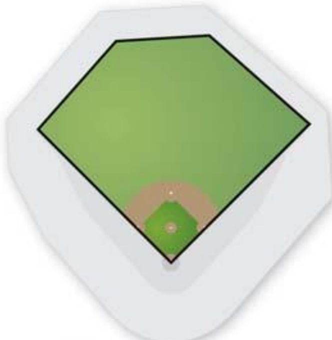
27. **CHALLENGE** Sides are added to a convex polygon so that the sum of its interior angle measures is increased by 540°. How many sides are added to the polygon? *Explain* your reasoning.

PROBLEM SOLVING

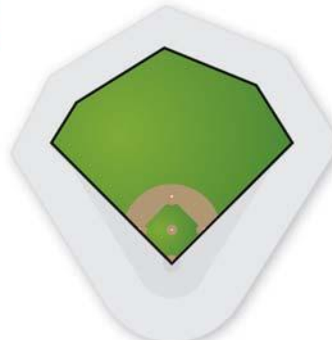
EXAMPLE 1
on p. 507
for Exs. 28–29

BASEBALL The outline of the playing field at a baseball park is a polygon, as shown. Find the sum of the measures of the interior angles of the polygon.

28.



29.



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EXAMPLE 5
on p. 510
for Exs. 30–31

30. JEWELRY BOX The base of a jewelry box is shaped like a regular hexagon. What is the measure of each interior angle of the hexagon?

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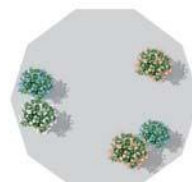
31. GREENHOUSE The floor of the greenhouse shown is a shaped like a regular decagon. Find the measure of an interior angle of the regular decagon. Then find the measure of an exterior angle.



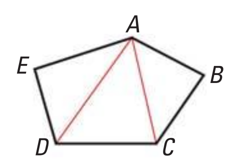
32. MULTI-STEP PROBLEM In pentagon *PQRST*, $\angle P$, $\angle Q$, and $\angle S$ are right angles, and $\angle R \cong \angle T$.

a. Draw a Diagram Sketch pentagon *PQRST*. Mark the right angles and the congruent angles.

b. Calculate Find the sum of the interior angle measures of *PQRST*.

c. Calculate Find $m\angle R$ and $m\angle T$.

33. PROVING THEOREM 8.1 FOR PENTAGONS The Polygon Interior Angles Theorem states that the sum of the measures of the interior angles of an n-gon is $(n - 2) \cdot 180°$. Write a paragraph proof of this theorem for the case when $n = 5$.



34. PROVING A COROLLARY Write a paragraph proof of the Corollary to the Polygon Interior Angles Theorem.

35. PROVING THEOREM 8.2 Use the plan below to write a paragraph proof of the Polygon Exterior Angles Theorem.

Plan for Proof In a convex n-gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180°. Multiply by n to get the sum of all such sums at each vertex. Then subtract the sum of the interior angles derived by using the Polygon Interior Angles Theorem.

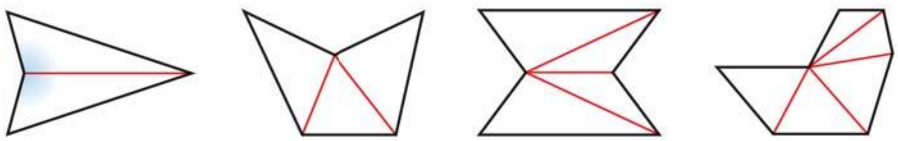
36. **MULTIPLE REPRESENTATIONS** The formula for the measure of each interior angle in a regular polygon can be written in function notation.

a. **Writing a Function** Write a function $h(n)$, where n is the number of sides in a regular polygon and $h(n)$ is the measure of any interior angle in the regular polygon.

b. **Using a Function** Use the function from part (a) to find $h(9)$. Then use the function to find n if $h(n) = 150°$.

c. **Graphing a Function** Graph the function from part (a) for $n = 3, 4, 5, 6, 7,$ and 8. Based on your graph, *describe* what happens to the value of $h(n)$ as n increases. *Explain* your reasoning.

37. ★ **EXTENDED RESPONSE** In a concave polygon, at least one interior angle measure is greater than 180°. For example, the measure of the shaded angle in the concave quadrilateral below is 210°.



a. In the diagrams above, the interiors of a concave quadrilateral, pentagon, hexagon, and heptagon are divided into triangles. Make a table like the one in the Activity on page 506. For each of the polygons shown above, record the number of sides, the number of triangles, and the sum of the measures of the interior angles.

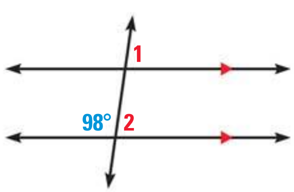
b. Write an algebraic expression that you can use to find the sum of the measures of the interior angles of a concave polygon. *Explain*.

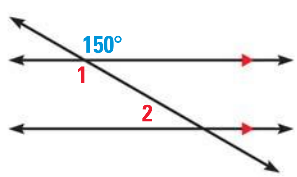
38. **CHALLENGE** Polygon *ABCDEFGH* is a regular octagon. Suppose sides $\overline{AB}$ and $\overline{CD}$ are extended to meet at a point *P*. Find $m\angle BPC$. *Explain* your reasoning. Include a diagram with your answer.

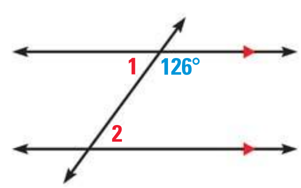
MIXED REVIEW

PREVIEW
Prepare for Lesson 8.2 in Exs. 39–41.

Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning. *(p. 154)*

39.


40.


41.


42. Quadrilaterals *JKLM* and *PQRS* are similar. If $JK = 3.6$ centimeters and $PQ = 1.2$ centimeters, find the scale factor of *JKLM* to *PQRS*. *(p. 372)*

43. Quadrilaterals *ABCD* and *EFGH* are similar. The scale factor of *ABCD* to *EFGH* is 8 : 5, and the perimeter of *ABCD* is 90 feet. Find the perimeter of *EFGH*. *(p. 372)*

Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree. *(p. 483)*

44. $\sin A = 0.77$
45. $\sin A = 0.35$
46. $\cos A = 0.81$
47. $\cos A = 0.23$

Investigating Geometry **ACTIVITY** *Use before Lesson 8.2*

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8.2 Investigate Parallelograms

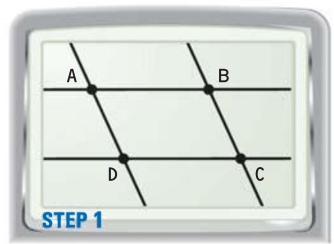
MATERIALS • graphing calculator or computer

QUESTION **What are some of the properties of a parallelogram?**

You can use geometry drawing software to investigate relationships in special quadrilaterals.

EXPLORE **Draw a quadrilateral**

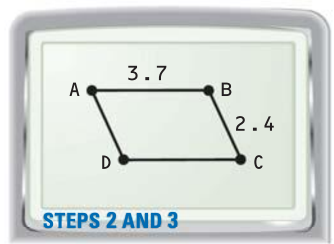
STEP 1 ***Draw parallel lines*** Construct $\overleftrightarrow{AB}$ and a line parallel to $\overleftrightarrow{AB}$ through point C. Then construct $\overleftrightarrow{BC}$ and a line parallel to $\overleftrightarrow{BC}$ through point A. Finally, construct a point D at the intersection of the line drawn parallel to $\overleftrightarrow{AB}$ and the line drawn parallel to $\overleftrightarrow{BC}$.



STEP 1

STEP 2 ***Draw quadrilateral*** Construct segments to form the sides of quadrilateral $ABCD$. After you construct $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, hide the parallel lines that you drew in Step 1.

STEP 3 ***Measure side lengths*** Measure the side lengths AB, BC, CD, and DA. Drag point A or point B to change the side lengths of $ABCD$. What do you notice about the side lengths?



STEPS 2 AND 3

STEP 4 ***Measure angles*** Find the measures of $\angle A$, $\angle B$, $\angle C$, and $\angle D$. Drag point A or point B to change the angle measures of $ABCD$. What do you notice about the angle measures?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. The quadrilateral you drew in the Explore is called a *parallelogram*. Why do you think this type of quadrilateral has this name?
2. Based on your observations, make a conjecture about the side lengths of a parallelogram and a conjecture about the angle measures of a parallelogram.
3. **REASONING** Draw a parallelogram and its diagonals. Measure the distance from the intersection of the diagonals to each vertex of the parallelogram. Make and test a conjecture about the diagonals of a parallelogram.

8.2 Use Properties of Parallelograms

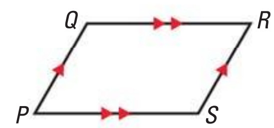
Before You used a property of polygons to find angle measures.

Now You will find angle and side measures in parallelograms.

Why? So you can solve a problem about airplanes, as in Ex. 38.

Key Vocabulary
- **parallelogram**

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. The term "parallelogram *PQRS*" can be written as ▱*PQRS*. In ▱*PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.



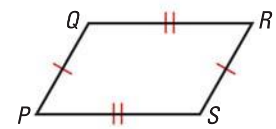
THEOREMS *For Your Notebook*

THEOREM 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If *PQRS* is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

Proof: p. 516

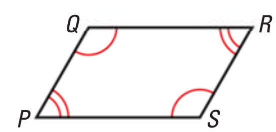


THEOREM 8.4

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

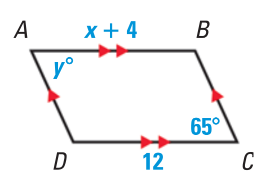
If *PQRS* is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

Proof: Ex. 42, p. 520



EXAMPLE 1 Use properties of parallelograms

xy ALGEBRA Find the values of *x* and *y*.



ABCD is a parallelogram by the definition of a parallelogram. Use Theorem 8.3 to find the value of *x*.

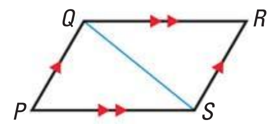
$AB = CD$	Opposite sides of a ▱ are ≅.
$x + 4 = 12$	Substitute $x + 4$ for AB and 12 for CD.
$x = 8$	Subtract 4 from each side.

By Theorem 8.4, $\angle A \cong \angle C$, or $m\angle A = m\angle C$. So, $y° = 65°$.

▸ In ▱*ABCD*, $x = 8$ and $y = 65$.

PROOF Theorem 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.



GIVEN ▶ $PQRS$ is a parallelogram.

PROVE ▶ $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$

Plan for Proof

a. Draw diagonal $\overline{QS}$ to form $\triangle PQS$ and $\triangle RSQ$.

b. Use the ASA Congruence Postulate to show that $\triangle PQS \cong \triangle RSQ$.

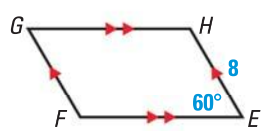
c. Use congruent triangles to show that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

Plan in Action

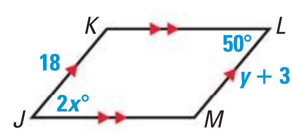
	STATEMENTS	REASONS
a.	1. $PQRS$ is a ▱.	1. Given
	2. Draw $\overline{QS}$.	2. Through any 2 points there exists exactly 1 line.
	3. $\overline{PQ} \parallel \overline{RS}$, $\overline{QR} \parallel \overline{PS}$	3. Definition of parallelogram
b.	4. $\angle PQS \cong \angle RSQ$, $\angle PSQ \cong \angle RQS$	4. Alternate Interior Angles Theorem
	5. $\overline{QS} \cong \overline{QS}$	5. Reflexive Property of Congruence
	6. $\triangle PQS \cong \triangle RSQ$	6. ASA Congruence Postulate
c.	7. $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$	7. Corresp. parts of $\cong$ △s are $\cong$.

✓ GUIDED PRACTICE for Example 1

1. Find FG and $m\angle G$.

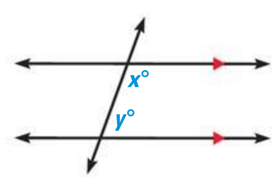


2. Find the values of x and y.



INTERIOR ANGLES The Consecutive Interior Angles Theorem (page 155) states that if two parallel lines are cut by a transversal, then the pairs of consecutive interior angles formed are supplementary.

A pair of consecutive angles in a parallelogram are like a pair of consecutive interior angles between parallel lines. This similarity suggests Theorem 8.5.



$x° + y° = 180°$

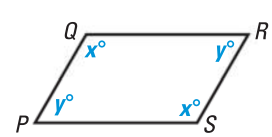
THEOREM *For Your Notebook*

THEOREM 8.5

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If $PQRS$ is a parallelogram, then $x° + y° = 180°$.

Proof: Ex. 43, p. 520



EXAMPLE 2 Use properties of a parallelogram

DESK LAMP As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110°$.

Solution

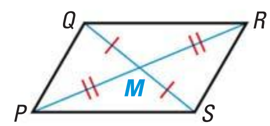
By Theorem 8.5, the consecutive angle pairs in $\square ABCD$ are supplementary. So, $m\angle ADC + m\angle BCD = 180°$. Because $m\angle ADC = 110°$, $m\angle BCD = 180° - 110° = 70°$.

THEOREM *For Your Notebook*

THEOREM 8.6

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Proof: Ex. 44, p. 521



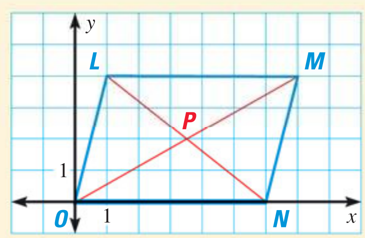
$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$

EXAMPLE 3 Standardized Test Practice

The diagonals of $\square LMNO$ intersect at point P. What are the coordinates of P?

Ⓐ $\left(\frac{7}{2}, 2\right)$ Ⓑ $\left(2, \frac{7}{2}\right)$

Ⓒ $\left(\frac{5}{2}, 2\right)$ Ⓓ $\left(2, \frac{5}{2}\right)$



Solution

By Theorem 8.6, the diagonals of a parallelogram bisect each other. So, P is the midpoint of diagonals $\overline{LN}$ and $\overline{OM}$. Use the Midpoint Formula.

Coordinates of midpoint P of $\overline{OM} = \left(\frac{7+0}{2}, \frac{4+0}{2}\right) = \left(\frac{7}{2}, 2\right)$

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

SIMPLIFY CALCULATIONS

In Example 3, you can use either diagonal to find the coordinates of P. Using $\overline{OM}$ simplifies calculations because one endpoint is (0, 0).

✓ GUIDED PRACTICE for Examples 2 and 3

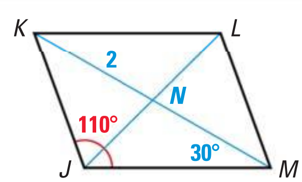
Find the indicated measure in $\square JKLM$.

3. NM

4. KM

5. $m\angle JML$

6. $m\angle KML$



8.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 29, 35, and 41

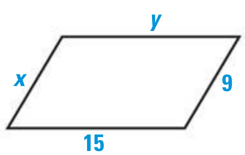
SKILL PRACTICE

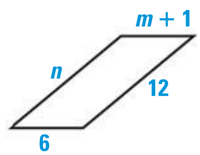
1. **VOCABULARY** What property of a parallelogram is included in the definition of a parallelogram? What properties are described by the theorems in this lesson?

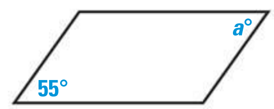
2. ★ **WRITING** In parallelogram $ABCD$, $m\angle A = 65°$. *Explain* how you would find the other angle measures of ▱$ABCD$.

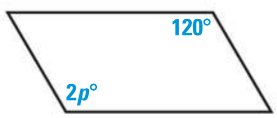
EXAMPLE 1 on p. 515 for Exs. 3–8

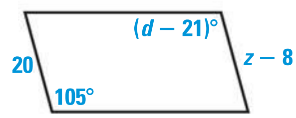
ALGEBRA **Find the value of each variable in the parallelogram.**

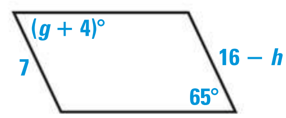
3. 

4. 

5. 

6. 

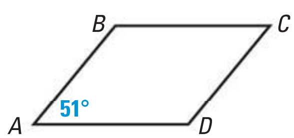
7. 

8. 

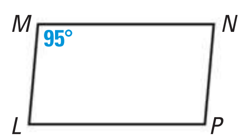
EXAMPLE 2 on p. 517 for Exs. 9–12

FINDING ANGLE MEASURES **Find the measure of the indicated angle in the parallelogram.**

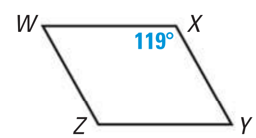
9. Find $m\angle B$.



10. Find $m\angle L$.



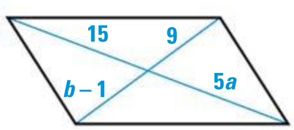
11. Find $m\angle Y$.

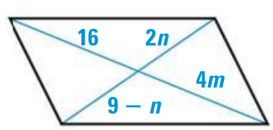


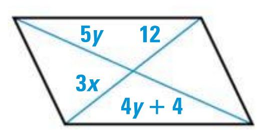
12. **SKETCHING** In ▱$PQRS$, $m\angle R$ is 24 degrees more than $m\angle S$. Sketch ▱$PQRS$. Find the measure of each interior angle. Then label each angle with its measure.

EXAMPLE 3 on p. 517 for Exs. 13–16

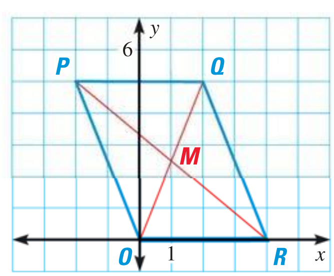
ALGEBRA **Find the value of each variable in the parallelogram.**

13. 

14. 

15. 

16. ★ **MULTIPLE CHOICE** The diagonals of parallelogram $OPQR$ intersect at point M. What are the coordinates of point M?



Ⓐ $\left(1, \frac{5}{2}\right)$　　Ⓑ $\left(2, \frac{5}{2}\right)$

Ⓒ $\left(1, \frac{3}{2}\right)$　　Ⓓ $\left(2, \frac{3}{2}\right)$

REASONING **Use the photo to copy and complete the statement. *Explain*.**

17. $\overline{AD} \cong$ __?__

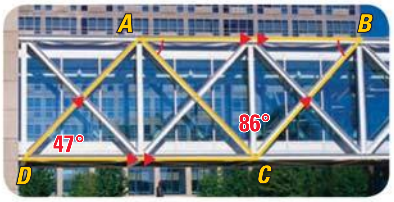
18. $\angle DAB \cong$ __?__

19. $\angle BCA \cong$ __?__

20. $m\angle ABC =$ __?__

21. $m\angle CAB =$ __?__

22. $m\angle CAD =$ __?__



USING A DIAGRAM **Find the indicated measure in ▱*EFGH*. *Explain*.**

23. $m\angle EJF$

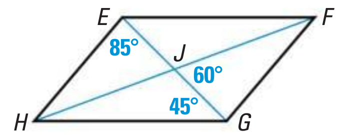
24. $m\angle EGF$

25. $m\angle HFG$

26. $m\angle GEF$

27. $m\angle HGF$

28. $m\angle EHG$



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29. ★ **MULTIPLE CHOICE** In parallelogram *ABCD*, $AB = 14$ inches and $BC = 20$ inches. What is the perimeter (in inches) of ▱*ABCD*?

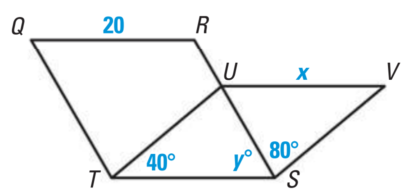
Ⓐ 28 Ⓑ 40 Ⓒ 68 Ⓓ 280

30. **ALGEBRA** The measure of one interior angle of a parallelogram is 0.25 times the measure of another angle. Find the measure of each angle.

31. **ALGEBRA** The measure of one interior angle of a parallelogram is 50 degrees more than 4 times the measure of another angle. Find the measure of each angle.

32. **ERROR ANALYSIS** In ▱*ABCD*, $m\angle B = 50°$. A student says that $m\angle A = 50°$. *Explain* why this statement is incorrect.

33. **USING A DIAGRAM** In the diagram, *QRST* and *STUV* are parallelograms. Find the values of *x* and *y*. *Explain* your reasoning.

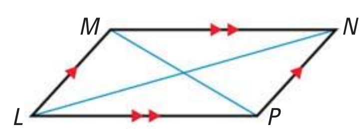


34. **FINDING A PERIMETER** The sides of ▱*MNPQ* are represented by the expressions below. Sketch ▱*MNPQ* and find its perimeter.

$MQ = -2x + 37$ $QP = y + 14$ $NP = x - 5$ $MN = 4y + 5$

35. ★ **SHORT RESPONSE** In *ABCD*, $m\angle B = 124°$, $m\angle A = 66°$, and $m\angle C = 124°$. *Explain* why *ABCD* cannot be a parallelogram.

36. **FINDING ANGLE MEASURES** In ▱*LMNP* shown at the right, $m\angle MLN = 32°$, $m\angle NLP = (x^2)°$, $m\angle MNP = 12x°$, and $\angle MNP$ is an acute angle. Find $m\angle NLP$.



37. **CHALLENGE** Points $A(1, 2)$, $B(3, 6)$, and $C(6, 4)$ are three vertices of ▱*ABCD*. Find the coordinates of each point that could be vertex *D*. Sketch each possible parallelogram in a separate coordinate plane. *Justify* your answers.

PROBLEM SOLVING

EXAMPLE 2
on p. 517
for Ex. 38

38. AIRPLANE The diagram shows the mechanism for opening the canopy on a small airplane. Two pivot arms attach at four pivot points A, B, C, and D. These points form the vertices of a parallelogram. Find $m\angle D$ when $m\angle C = 40°$. *Explain* your reasoning.

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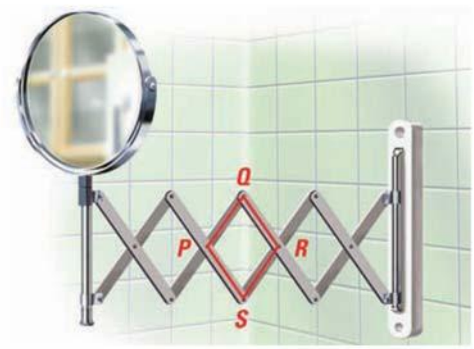
39. MIRROR The mirror shown is attached to the wall by an arm that can extend away from the wall. In the figure, points P, Q, R, and S are the vertices of a parallelogram. This parallelogram is one of several that change shape as the mirror is extended.



a. If $PQ = 3$ inches, find RS.

b. If $m\angle Q = 70°$, what is $m\angle S$?

c. What happens to $m\angle P$ as $m\angle Q$ increases? What happens to QS as $m\angle Q$ decreases? *Explain*.

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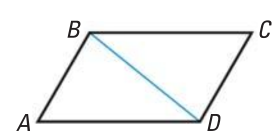
40. USING RATIOS In $\square LMNO$, the ratio of LM to MN is 4 : 3. Find LM if the perimeter of $LMNO$ is 28.

41. ★ OPEN-ENDED MATH Draw a triangle. Copy the triangle and combine the two triangles to form a quadrilateral. Show that the quadrilateral is a parallelogram. Then show how you can make additional copies of the triangle to form a larger parallelogram that is similar to the first parallelogram. *Justify* your method.

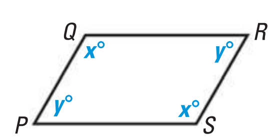
42. PROVING THEOREM 8.4 Use the diagram of quadrilateral $ABCD$ with the auxiliary line segment drawn to write a two-column proof of Theorem 8.4.



GIVEN ▶ $ABCD$ is a parallelogram.

PROVE ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$

43. PROVING THEOREM 8.5 Use properties of parallel lines to prove Theorem 8.5.



GIVEN ▶ $PQRS$ is a parallelogram.

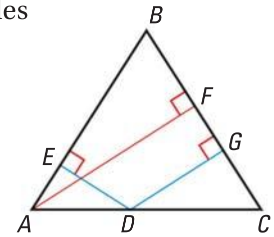
PROVE ▶ $x° + y° = 180°$

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

44. **PROVING THEOREM 8.6** Theorem 8.6 states that if a quadrilateral is a parallelogram, then its diagonals bisect each other. Write a two-column proof of Theorem 8.6.

45. **CHALLENGE** Suppose you choose a point on the base of an isosceles triangle. You draw segments from that point perpendicular to the legs of the triangle. Prove that the sum of the lengths of those segments is equal to the length of the altitude drawn to one leg.



GIVEN ▶ $\triangle ABC$ is isosceles with base $\overline{AC}$,
$\overline{AF}$ is the altitude drawn to $\overline{BC}$,
$\overline{DE} \perp \overline{AB}$, $\overline{DG} \perp \overline{BC}$

PROVE ▶ For D anywhere on $\overline{AC}$, $DE + DG = AF$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.3 in Exs. 46–48.

Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer. *(p. 171)*

46. **Line 1:** (2, 4), (4, 1)
Line 2: (5, 7), (9, 0)

47. **Line 1:** (−6, 7), (−2, 3)
Line 2: (9, −1), (2, 6)

48. **Line 1:** (−3, 0), (−6, 5)
Line 2: (3, −5), (5, −10)

Decide if the side lengths form a triangle. If so, would the triangle be *acute*, *right*, or *obtuse*? *(p. 441)*

49. 9, 13, and 6

50. 10, 12, and 7

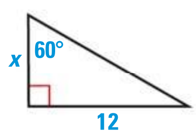
51. 5, 9, and $\sqrt{106}$

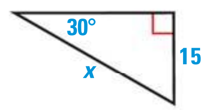
52. 8, 12, and 4

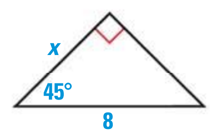
53. 24, 10, and 26

54. 9, 10, and 11

Find the value of *x*. Write your answer in simplest radical form. *(p. 457)*

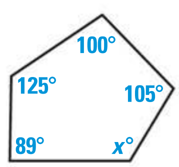
55.


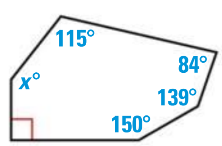
56.


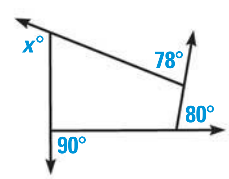
57.


QUIZ *for Lessons 8.1–8.2*

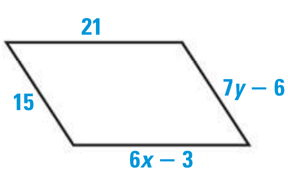
Find the value of *x*. *(p. 507)*

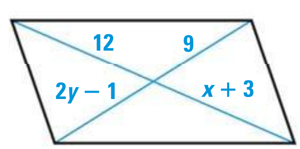
1.


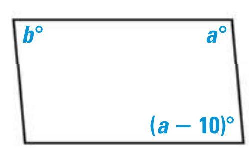
2.


3.


Find the value of each variable in the parallelogram. *(p. 515)*

4.


5.


6.


8.3 Show that a Quadrilateral is a Parallelogram

Before You identified properties of parallelograms.

Now You will use properties to identify parallelograms.

Why? So you can describe how a music stand works, as in Ex. 32.

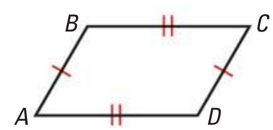
Key Vocabulary
- **parallelogram,** *p. 515*

Given a parallelogram, you can use Theorem 8.3 and Theorem 8.4 to prove statements about the angles and sides of the parallelogram. The converses of Theorem 8.3 and Theorem 8.4 are stated below. You can use these and other theorems in this lesson to prove that a quadrilateral with certain properties is a parallelogram.

THEOREMS — *For Your Notebook*

THEOREM 8.7

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

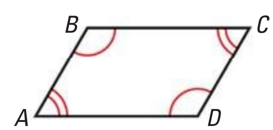


If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof: below

THEOREM 8.8

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



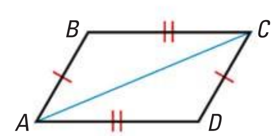
If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof: Ex. 38, p. 529

PROOF Theorem 8.7

GIVEN ▶ $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$

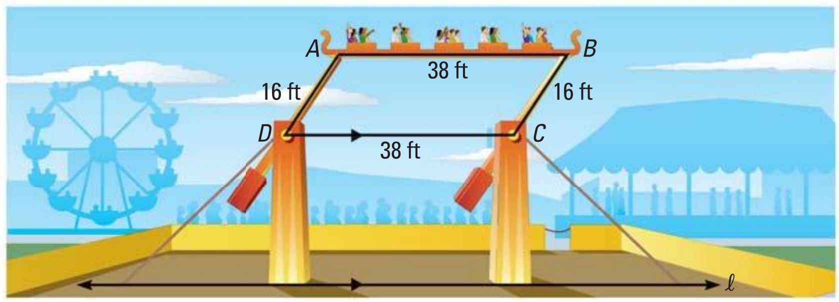
PROVE ▶ $ABCD$ is a parallelogram.



Proof Draw $\overline{AC}$, forming $\triangle ABC$ and $\triangle CDA$. You are given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Also, $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. So, $\triangle ABC \cong \triangle CDA$ by the SSS Congruence Postulate. Because corresponding parts of congruent triangles are congruent, $\angle BAC \cong \angle DCA$ and $\angle BCA \cong DAC$. Then, by the Alternate Interior Angles Converse, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. By definition, $ABCD$ is a parallelogram.

EXAMPLE 1 Solve a real-world problem

RIDE An amusement park ride has a moving platform attached to four swinging arms. The platform swings back and forth, higher and higher, until it goes over the top and around in a circular motion. In the diagram below, $\overline{AD}$ and $\overline{BC}$ represent two of the swinging arms, and $\overline{DC}$ is parallel to the ground (line ℓ). *Explain* why the moving platform $\overline{AB}$ is always parallel to the ground.



Solution

The shape of quadrilateral *ABCD* changes as the moving platform swings around, but its side lengths do not change. Both pairs of opposite sides are congruent, so *ABCD* is a parallelogram by Theorem 8.7.

By the definition of a parallelogram, $\overline{AB} \parallel \overline{DC}$. Because $\overline{DC}$ is parallel to line ℓ, $\overline{AB}$ is also parallel to line ℓ by the Transitive Property of Parallel Lines. So, the moving platform is parallel to the ground.

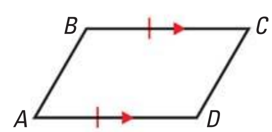
✓ GUIDED PRACTICE for Example 1

1. In quadrilateral *WXYZ*, $m\angle W = 42°$, $m\angle X = 138°$, $m\angle Y = 42°$. Find $m\angle Z$. Is *WXYZ* a parallelogram? *Explain* your reasoning.

THEOREMS *For Your Notebook*

THEOREM 8.9

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

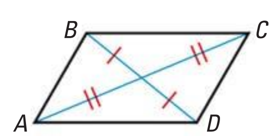


If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then *ABCD* is a parallelogram.

Proof: Ex. 33, p. 528

THEOREM 8.10

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



If $\overline{BD}$ and $\overline{AC}$ bisect each other, then *ABCD* is a parallelogram.

Proof: Ex. 39, p. 529

EXAMPLE 2 Identify a parallelogram

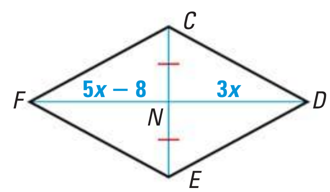
ARCHITECTURE The doorway shown is part of a building in England. Over time, the building has leaned sideways. *Explain* how you know that $SV = TU$.

Solution

In the photograph, $\overline{ST} \parallel \overline{UV}$ and $\overline{ST} \cong \overline{UV}$. By Theorem 8.9, quadrilateral $STUV$ is a parallelogram. By Theorem 8.3, you know that opposite sides of a parallelogram are congruent. So, $SV = TU$.

EXAMPLE 3 Use algebra with parallelograms

ALGEBRA For what value of x is quadrilateral *CDEF* a parallelogram?



Solution

By Theorem 8.10, if the diagonals of $CDEF$ bisect each other, then it is a parallelogram. You are given that $\overline{CN} \cong \overline{EN}$. Find x so that $\overline{FN} \cong \overline{DN}$.

$FN = DN$	Set the segment lengths equal.
$5x - 8 = 3x$	Substitute $5x - 8$ for FN and $3x$ for DN.
$2x - 8 = 0$	Subtract $3x$ from each side.
$2x = 8$	Add 8 to each side.
$x = 4$	Divide each side by 2.

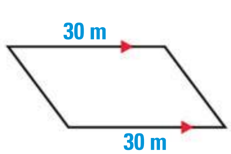
When $x = 4$, $FN = 5(4) - 8 = 12$ and $DN = 3(4) = 12$.

▶ Quadrilateral $CDEF$ is a parallelogram when $x = 4$.

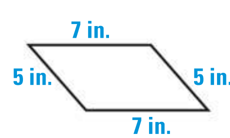
✓ GUIDED PRACTICE for Examples 2 and 3

What theorem can you use to show that the quadrilateral is a parallelogram?

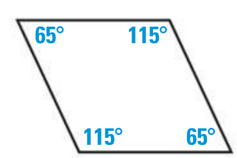
2.



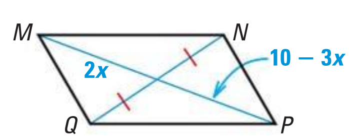
3.



4.

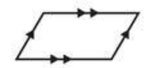


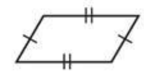
5. For what value of x is quadrilateral $MNPQ$ a parallelogram? *Explain* your reasoning.

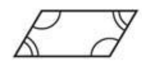


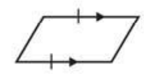
CONCEPT SUMMARY *For Your Notebook*

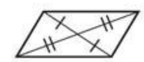
Ways to Prove a Quadrilateral is a Parallelogram

1. Show both pairs of opposite sides are parallel.
   (DEFINITION)
   

2. Show both pairs of opposite sides are congruent.
   (THEOREM 8.7)
   

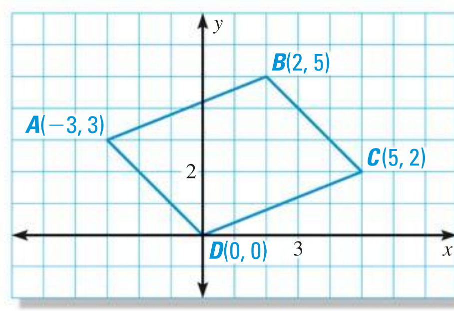
3. Show both pairs of opposite angles are congruent.
   (THEOREM 8.8)
   

4. Show one pair of opposite sides are congruent and parallel.
   (THEOREM 8.9)
   

5. Show the diagonals bisect each other.
   (THEOREM 8.10)
   

EXAMPLE 4 Use coordinate geometry

Show that quadrilateral *ABCD* is a parallelogram.



ANOTHER WAY
For alternative methods for solving the problem in Example 4, turn to page 530 for the **Problem Solving Workshop**.

Solution

One way is to show that a pair of sides are congruent and parallel. Then apply Theorem 8.9.

First use the Distance Formula to show that $\overline{AB}$ and $\overline{CD}$ are congruent.

$$AB = \sqrt{[2-(-3)]^2 + (5-3)^2} = \sqrt{29} \qquad CD = \sqrt{(5-0)^2 + (2-0)^2} = \sqrt{29}$$

Because $AB = CD = \sqrt{29}$, $\overline{AB} \cong \overline{CD}$.

Then use the slope formula to show that $\overline{AB} \parallel \overline{CD}$.

$$\text{Slope of } \overline{AB} = \frac{5-(3)}{2-(-3)} = \frac{2}{5} \qquad \text{Slope of } \overline{CD} = \frac{2-0}{5-0} = \frac{2}{5}$$

Because $\overline{AB}$ and $\overline{CD}$ have the same slope, they are parallel.

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel. So, *ABCD* is a parallelogram by Theorem 8.9.

✓ GUIDED PRACTICE for Example 4

6. Refer to the Concept Summary above. *Explain* how other methods can be used to show that quadrilateral *ABCD* in Example 4 is a parallelogram.

8.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 11, and 31

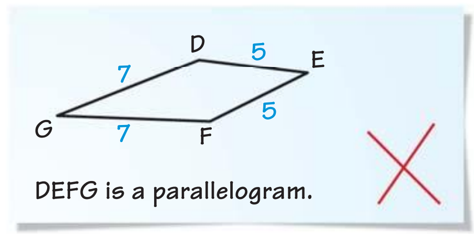
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 18, and 37

SKILL PRACTICE

1. **VOCABULARY** *Explain* how knowing that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ allows you to show that quadrilateral *ABCD* is a parallelogram.

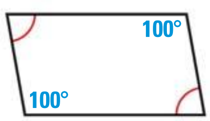
2. ★ **WRITING** A quadrilateral has four congruent sides. Is the quadrilateral a parallelogram? *Justify* your answer.

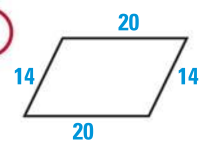
3. **ERROR ANALYSIS** A student claims that because two pairs of sides are congruent, quadrilateral *DEFG* shown at the right is a parallelogram. *Describe* the error that the student is making.

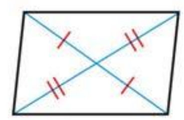


EXAMPLES 1 and 2 on pp. 523–524 for Exs. 4–7

REASONING **What theorem can you use to show that the quadrilateral is a parallelogram?**

4.


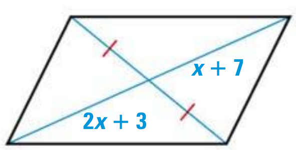
5.


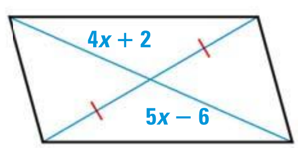
6.


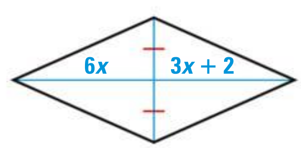
7. ★ **SHORT RESPONSE** When you shift gears on a bicycle, a mechanism called a *derailleur* moves the chain to a new gear. For the derailleur shown below, $JK = 5.5$ cm, $KL = 2$ cm, $ML = 5.5$ cm, and $MJ = 2$ cm. *Explain* why $\overline{JK}$ and $\overline{ML}$ are always parallel as the derailleur moves.

EXAMPLE 3 on p. 524 for Exs. 8–10

ALGEBRA **For what value of *x* is the quadrilateral a parallelogram?**

8.


9.


10.


EXAMPLE 4 on p. 525 for Exs. 11–14

COORDINATE GEOMETRY **The vertices of quadrilateral *ABCD* are given. Draw *ABCD* in a coordinate plane and show that it is a parallelogram.**

11. $A(0, 1), B(4, 4), C(12, 4), D(8, 1)$

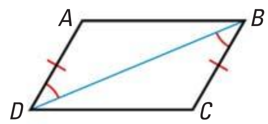
12. $A(-3, 0), B(-3, 4), C(3, -1), D(3, -5)$

13. $A(-2, 3), B(-5, 7), C(3, 6), D(6, 2)$

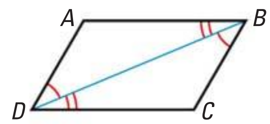
14. $A(-5, 0), B(0, 4), C(3, 0), D(-2, -4)$

REASONING ***Describe*** **how to prove that *ABCD* is a parallelogram.**

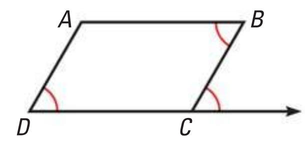
15.



16.



17.



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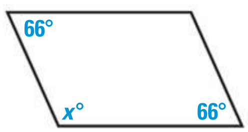
18. ★ **MULTIPLE CHOICE** In quadrilateral $WXYZ$, $\overline{WZ}$ and $\overline{XY}$ are congruent and parallel. Which statement below is not necessarily true?

Ⓐ $m\angle Y + m\angle W = 180°$ Ⓑ $\angle X \cong \angle Z$

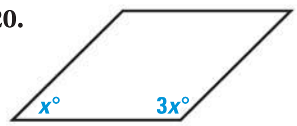
Ⓒ $\overline{WX} \cong \overline{ZY}$ Ⓓ $\overline{WX} \parallel \overline{ZY}$

ALGEBRA For what value of *x* is the quadrilateral a parallelogram?

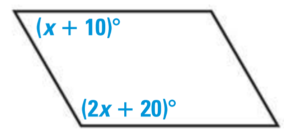
19.



20.



21.

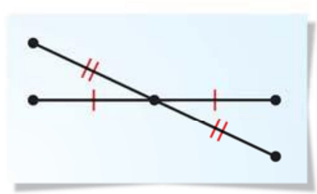


BICONDITIONALS Write the indicated theorems as a biconditional statement.

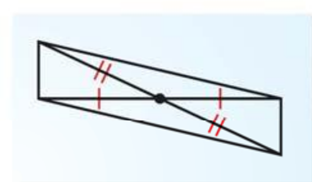
22. Theorem 8.3, page 515 and Theorem 8.7, page 522

23. Theorem 8.4, page 515 and Theorem 8.8, page 522

24. **REASONING** Follow the steps below to draw a parallelogram. *Explain* why this method works. State a theorem to support your answer.



STEP 1 Use a ruler to draw two segments that intersect at their midpoints.



STEP 2 Connect the endpoints of the segments to form a quadrilateral.

COORDINATE GEOMETRY Three of the vertices of ▱*ABCD* are given. Find the coordinates of point *D*. Show your method.

25. $A(-2, -3)$, $B(4, -3)$, $C(3, 2)$, $D(x, y)$

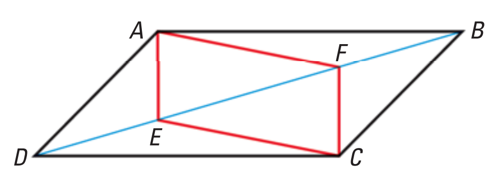
26. $A(-4, 1)$, $B(-1, 5)$, $C(6, 5)$, $D(x, y)$

27. $A(-4, 4)$, $B(4, 6)$, $C(3, -1)$, $D(x, y)$

28. $A(-1, 0)$, $B(0, -4)$, $C(8, -6)$, $D(x, y)$

29. **CONSTRUCTION** There is more than one way to use a compass and a straightedge to construct a parallelogram. *Describe* a method that uses Theorem 8.7 or Theorem 8.9. Then use your method to construct a parallelogram.

30. **CHALLENGE** In the diagram, $ABCD$ is a parallelogram, $BF = DE = 12$, and $CF = 8$. Find AE. *Explain* your reasoning.



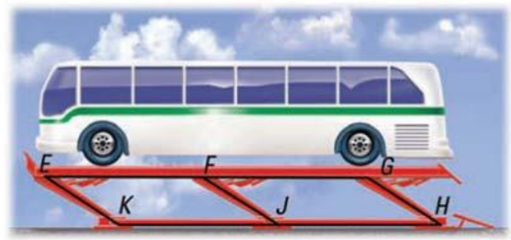
PROBLEM SOLVING

EXAMPLES 1 and 2
on pp. 523–524
for Exs. 31–32

31. **AUTOMOBILE REPAIR** The diagram shows an automobile lift. A bus drives on to the ramp ($\overline{EG}$). Levers ($\overline{EK}$, $\overline{FJ}$, and $\overline{GH}$) raise the bus. In the diagram, $\overline{EG} \cong \overline{KH}$ and $EK = FJ = GH$. Also, F is the midpoint of $\overline{EG}$, and J is the midpoint of $\overline{KH}$.

a. Identify all the quadrilaterals in the automobile lift. *Explain* how you know that each one is a parallelogram.

b. *Explain* why $\overline{EG}$ is always parallel to $\overline{KH}$.



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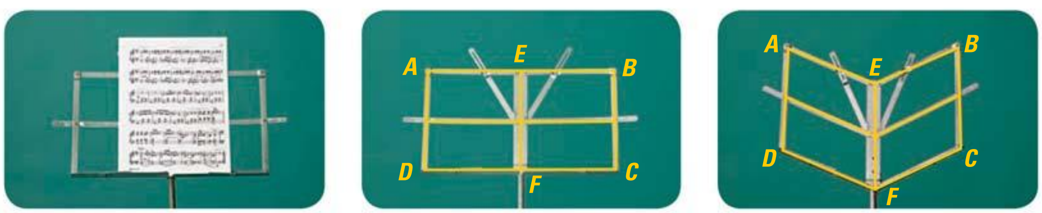
32. **MUSIC STAND** A music stand can be folded up, as shown below. In the diagrams, $\angle A \cong \angle EFD$, $\angle D \cong \angle AEF$, $\angle C \cong \angle BEF$, and $\angle B \cong \angle CFE$. *Explain* why $\overline{AD}$ and $\overline{BC}$ remain parallel as the stand is folded up. Which other labeled segments remain parallel?



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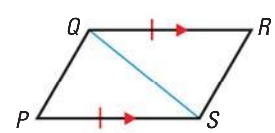
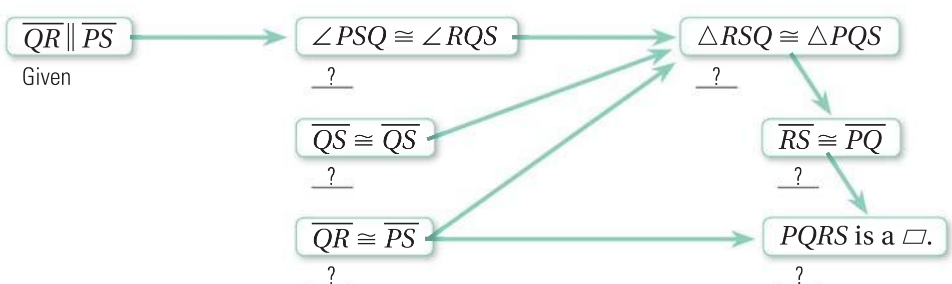
33. **PROVING THEOREM 8.9** Use the diagram of $PQRS$ with the auxiliary line segment drawn. Copy and complete the flow proof of Theorem 8.9.



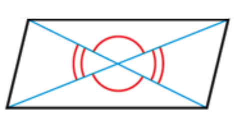
GIVEN ▶ $\overline{QR} \parallel \overline{PS}$, $\overline{QR} \cong \overline{PS}$

PROVE ▶ $PQRS$ is a parallelogram.

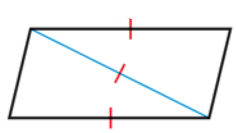


REASONING **A student claims incorrectly that the marked information can be used to show that the figure is a parallelogram. Draw a quadrilateral with the marked properties that is clearly *not* a parallelogram. *Explain*.**

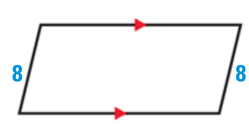
34.



35.



36.

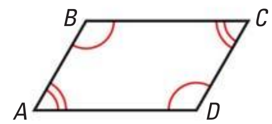


○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

37. ★ **EXTENDED RESPONSE** Theorem 8.5 states that if a quadrilateral is a parallelogram, then its consecutive angles are supplementary. Write the converse of Theorem 8.5. Then write a plan for proving the converse of Theorem 8.5. Include a diagram.

38. **PROVING THEOREM 8.8** Prove Theorem 8.8.

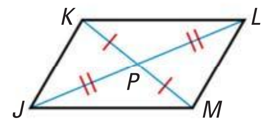


GIVEN ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$

PROVE ▶ *ABCD* is a parallelogram.

Hint: Let $x°$ represent $m\angle A$ and $m\angle C$, and let $y°$ represent $m\angle B$ and $m\angle D$. Write and simplify an equation involving x and y.

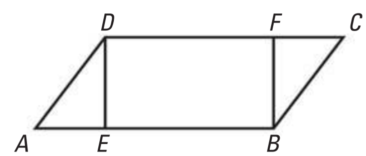
39. **PROVING THEOREM 8.10** Prove Theorem 8.10.



GIVEN ▶ Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other.

PROVE ▶ *JKLM* is a parallelogram.

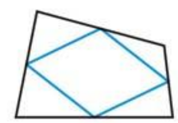
40. **PROOF** Use the diagram at the right.



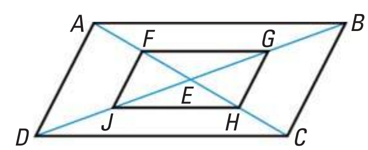
GIVEN ▶ *DEBF* is a parallelogram, $AE = CF$

PROVE ▶ *ABCD* is a parallelogram.

41. **REASONING** In the diagram, the midpoints of the sides of a quadrilateral have been joined to form what appears to be a parallelogram. Show that a quadrilateral formed by connecting the midpoints of the sides of any quadrilateral is *always* a parallelogram. (*Hint:* Draw a diagram. Include a diagonal of the larger quadrilateral. Show how two sides of the smaller quadrilateral are related to the diagonal.)



42. **CHALLENGE** Show that if *ABCD* is a parallelogram with its diagonals intersecting at *E*, then you can connect the midpoints *F*, *G*, *H*, and *J* of $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$, respectively, to form another parallelogram, *FGHJ*.



MIXED REVIEW

PREVIEW
Prepare for Lesson 8.4 in Exs. 43–45.

In Exercises 43–45, draw a figure that fits the description. *(p. 42)*

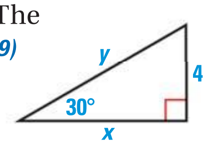
43. A quadrilateral that is equilateral but not equiangular

44. A quadrilateral that is equiangular but not equilateral

45. A quadrilateral that is concave

46. The width of a rectangle is 4 centimeters less than its length. The perimeter of the rectangle is 42 centimeters. Find its area. *(p. 49)*

47. Find the values of x and y in the triangle shown at the right. Write your answers in simplest radical form. *(p. 457)*



PROBLEM SOLVING WORKSHOP
LESSON 8.3

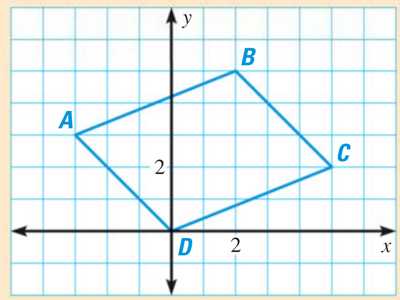
Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 525

MULTIPLE REPRESENTATIONS In Example 4 on page 525, the problem is solved by showing that one pair of opposite sides are congruent and parallel using the Distance Formula and the slope formula. There are other ways to show that a quadrilateral is a parallelogram.

PROBLEM

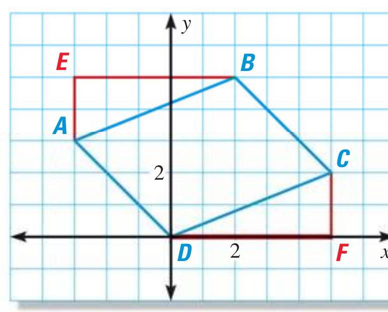
Show that quadrilateral *ABCD* is a parallelogram.



METHOD 1

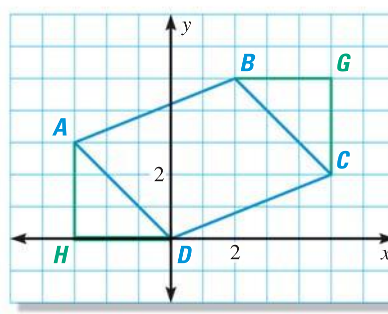
Use Opposite Sides You can show that both pairs of opposite sides are congruent.

STEP 1 **Draw** two right triangles. Use $\overline{AB}$ as the hypotenuse of $\triangle AEB$ and $\overline{CD}$ as the hypotenuse of $\triangle CFD$.



STEP 2 **Show** that $\triangle AEB \cong \triangle CFD$. From the graph, $AE = 2$, $BE = 5$, and $\angle E$ is a right angle. Similarly, $CF = 2$, $DF = 5$, and $\angle F$ is a right angle. So, $\triangle AEB \cong \triangle CFD$ by the SAS Congruence Postulate.

STEP 3 **Use** the fact that corresponding parts of congruent triangles are congruent to show that $\overline{AB} \cong \overline{CD}$.



STEP 4 **Repeat** Steps 1–3 for sides $\overline{AD}$ and $\overline{BC}$. You can prove that $\triangle AHD \cong \triangle CGB$. So, $\overline{AD} \cong \overline{CB}$.

▶ The pairs of opposite sides, $\overline{AB}$ and $\overline{CD}$ and $\overline{AD}$ and $\overline{CB}$, are congruent. So, *ABCD* is a parallelogram by Theorem 8.7.

METHOD 2 **Use Diagonals** You can show that the diagonals bisect each other.

STEP 1 **Use** the Midpoint Formula to find the midpoint of diagonal $\overline{AC}$.

The coordinates of the endpoints of $\overline{AC}$ are $A(-3, 3)$ and $C(5, 2)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 5}{2}, \frac{3 + 2}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = \left(1, \frac{5}{2}\right)$$

STEP 2 **Use** the Midpoint Formula to find the midpoint of diagonal $\overline{BD}$.

The coordinates of the endpoints of $\overline{BD}$ are $B(2, 5)$ and $D(0, 0)$.

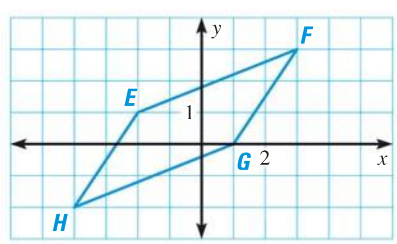
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 0}{2}, \frac{5 + 0}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = M\left(1, \frac{5}{2}\right)$$

▶ Because the midpoints of both diagonals are the same point, the diagonals bisect each other. So, *ABCD* is a parallelogram by Theorem 8.10.

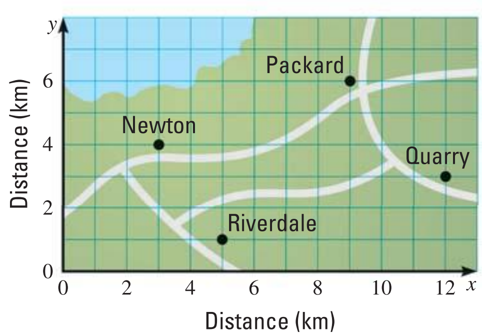
PRACTICE

1. **SLOPE** Show that quadrilateral *ABCD* in the problem on page 530 is a parallelogram by showing that both pairs of opposite sides are parallel.

2. **PARALLELOGRAMS** Use two methods to show that *EFGH* is a parallelogram.



3. **MAP** Do the four towns on the map form the vertices of a parallelogram? *Explain*.



4. **QUADRILATERALS** Is the quadrilateral a parallelogram? *Justify* your answer.

   a. $A(1, 0)$, $B(5, 0)$, $C(7, 2)$, $D(3, 2)$

   b. $E(3, 4)$ $F(9, 5)$, $G(6, 8)$, $H(6, 0)$

   c. $J(-1, 0)$, $K(2, -2)$, $L(2, 2)$, $M(-1, 4)$

5. **ERROR ANALYSIS** Quadrilateral *PQRS* has vertices $P(2, 2)$, $Q(3, 4)$, $R(6, 5)$, and $S(5, 3)$. A student makes the conclusion below. *Describe* and correct the error(s) made by the student.

> $\overline{PQ}$ and $\overline{QR}$ are opposite sides, so they should be congruent.
>
> $PQ = \sqrt{(3 - 2)^2 + (4 - 2)^2} = \sqrt{5}$
>
> $QR = \sqrt{(6 - 3)^2 + (5 - 4)^2} = \sqrt{10}$
>
> But $\overline{PQ} \ncong \overline{QR}$. So, PQRS is not a parallelogram.

6. **WRITING** Points $O(0, 0)$, $P(3, 5)$, and $Q(4, 0)$ are vertices of $\triangle OPQ$, and are also vertices of a parallelogram. Find all points *R* that could be the other vertex of the parallelogram. *Explain* your reasoning.

MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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Lessons 8.1–8.3

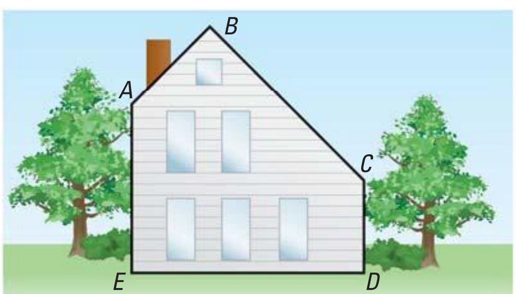
1. **MULTI-STEP PROBLEM** The shape of Iowa can be approximated by a polygon, as shown.

   a. How many sides does the polygon have? Classify the polygon.

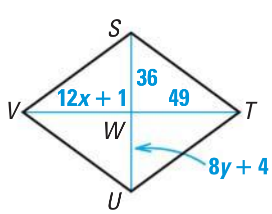
   b. What is the sum of the measures of the interior angles of the polygon?

   c. What is the sum of the measures of the exterior angles of the polygon?

2. **SHORT RESPONSE** A graphic designer is creating an electronic image of a house. In the drawing, $\angle B$, $\angle D$, and $\angle E$ are right angles, and $\angle A \cong \angle C$. *Explain* how to find $m\angle A$ and $m\angle C$.



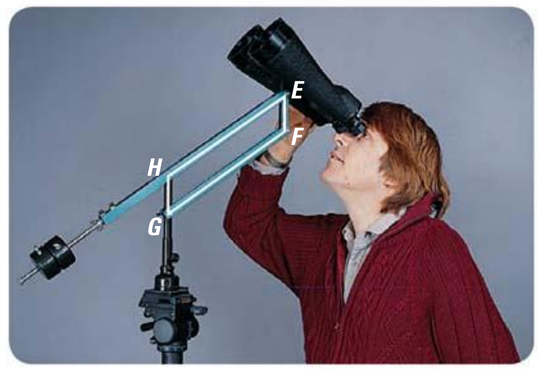
3. **SHORT RESPONSE** Quadrilateral *STUV* shown below is a parallelogram. Find the values of *x* and *y*. *Explain* your reasoning.



4. **GRIDDED ANSWER** A convex decagon has interior angles with measures 157°, 128°, 115°, 162°, 169°, 131°, 155°, 168°, x°, and $2x$°. Find the value of x.

5. **SHORT RESPONSE** The measure of an angle of a parallelogram is 12 degrees less than 3 times the measure of an adjacent angle. Explain how to find the measures of all the interior angles of the parallelogram.

6. **EXTENDED RESPONSE** A stand to hold binoculars in place uses a quadrilateral in its design. Quadrilateral *EFGH* shown below changes shape as the binoculars are moved. In the photograph, $\overline{EF}$ and $\overline{GH}$ are congruent and parallel.



   a. *Explain* why $\overline{EF}$ and $\overline{GH}$ remain parallel as the shape of *EFGH* changes. *Explain* why $\overline{EH}$ and $\overline{FG}$ remain parallel.

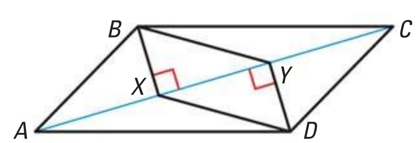
   b. As *EFGH* changes shape, $m\angle E$ changes from 55° to 50°. *Describe* how $m\angle F$, $m\angle G$, and $m\angle H$ will change. *Explain*.

7. **EXTENDED RESPONSE** The vertices of quadrilateral *MNPQ* are $M(-8, 1)$, $N(3, 4)$, $P(7, -1)$, and $Q(-4, -4)$.

   a. Use what you know about slopes of lines to prove that *MNPQ* is a parallelogram. *Explain* your reasoning.

   b. Use the Distance Formula to show that *MNPQ* is a parallelogram. *Explain*.

8. **EXTENDED RESPONSE** In $\square ABCD$, $\overline{BX} \perp \overline{AC}$, $\overline{DY} \perp \overline{AC}$. Show that *XBYD* is a parallelogram.



8.4 Properties of Rhombuses, Rectangles, and Squares

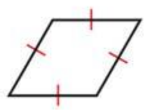
Before You used properties of parallelograms.

Now You will use properties of rhombuses, rectangles, and squares.

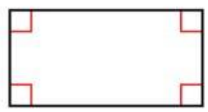
Why? So you can solve a carpentry problem, as in Example 4.

Key Vocabulary
- **rhombus**
- **rectangle**
- **square**

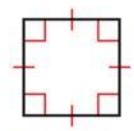
In this lesson, you will learn about three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.



A rhombus is a parallelogram with four congruent sides.



A rectangle is a parallelogram with four right angles.



A square is a parallelogram with four congruent sides and four right angles.

You can use the corollaries below to prove that a quadrilateral is a rhombus, rectangle, or square, without first proving that the quadrilateral is a parallelogram.

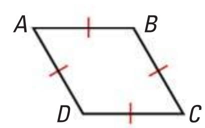
COROLLARIES — *For Your Notebook*

Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof: Ex. 57, p. 539

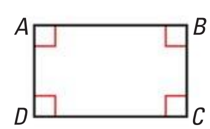


Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof: Ex. 58, p. 539

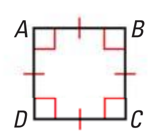


Square Corollary

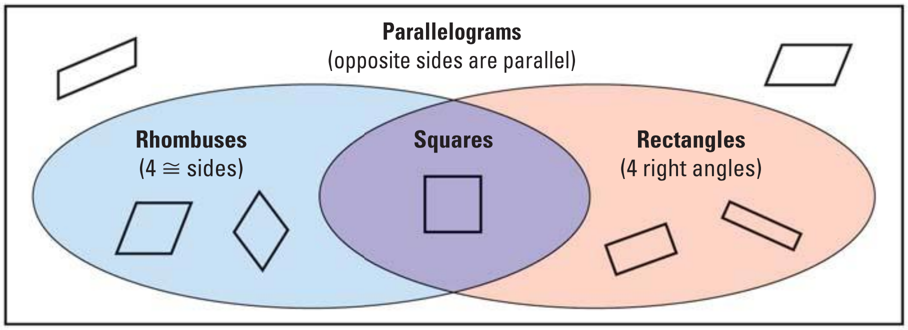
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof: Ex. 59, p. 539



The *Venn diagram* below illustrates some important relationships among parallelograms, rhombuses, rectangles, and squares. For example, you can see that a square is a rhombus because it is a parallelogram with four congruent sides. Because it has four right angles, a square is also a rectangle.



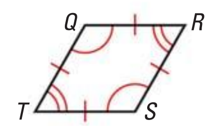
EXAMPLE 1 Use properties of special quadrilaterals

For any rhombus *QRST*, decide whether the statement is *always* or *sometimes* true. Draw a sketch and explain your reasoning.

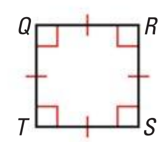
a. $\angle Q \cong \angle S$ **b.** $\angle Q \cong \angle R$

Solution

a. By definition, a rhombus is a parallelogram with four congruent sides. By Theorem 8.4, opposite angles of a parallelogram are congruent. So, $\angle Q \cong \angle S$. The statement is *always* true.

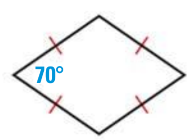


b. If rhombus *QRST* is a square, then all four angles are congruent right angles. So, $\angle Q \cong \angle R$ if *QRST* is a square. Because not all rhombuses are also squares, the statement is *sometimes* true.



EXAMPLE 2 Classify special quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



Solution

The quadrilateral has four congruent sides. One of the angles is not a right angle, so the rhombus is not also a square. By the Rhombus Corollary, the quadrilateral is a rhombus.

✓ GUIDED PRACTICE for Examples 1 and 2

1. For any rectangle *EFGH*, is it *always* or *sometimes* true that $\overline{FG} \cong \overline{GH}$? *Explain* your reasoning.

2. A quadrilateral has four congruent sides and four congruent angles. Sketch the quadrilateral and classify it.

DIAGONALS The theorems below describe some properties of the diagonals of rhombuses and rectangles.

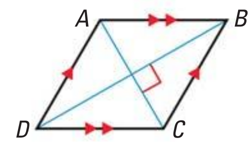
THEOREMS *For Your Notebook*

THEOREM 8.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

▱*ABCD* is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof: p. 536; Ex. 56, p. 539

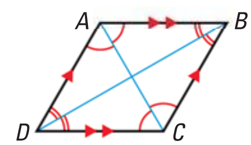


THEOREM 8.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

▱*ABCD* is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$ and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof: Exs. 60–61, p. 539

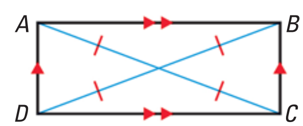


THEOREM 8.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

▱*ABCD* is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

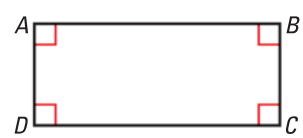
Proof: Exs. 63–64, p. 540



EXAMPLE 3 List properties of special parallelograms

Sketch rectangle *ABCD*. List everything that you know about it.

Solution



By definition, you need to draw a figure with the following properties:

- The figure is a parallelogram.
- The figure has four right angles.

Because *ABCD* is a parallelogram, it also has these properties:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent. Consecutive angles are supplementary.
- Diagonals bisect each other.

By Theorem 8.13, the diagonals of *ABCD* are congruent.

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✓ GUIDED PRACTICE for Example 3

3. Sketch square *PQRS*. List everything you know about the square.

BICONDITIONALS Recall that biconditionals such as Theorem 8.11 can be rewritten as two parts. To prove a biconditional, you must prove both parts.

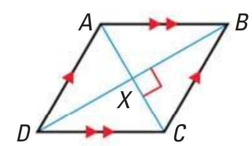
Conditional statement If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Converse If a parallelogram is a rhombus, then its diagonals are perpendicular.

PROOF Part of Theorem 8.11

PROVE THEOREMS
You will prove the other part of Theorem 8.11 in Exercise 56 on page 539.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.



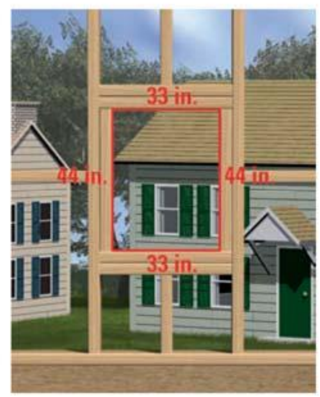
GIVEN ▶ *ABCD* is a parallelogram; $\overline{AC} \perp \overline{BD}$

PROVE ▶ *ABCD* is a rhombus.

Proof *ABCD* is a parallelogram, so $\overline{AC}$ and $\overline{BD}$ bisect each other, and $\overline{BX} \cong \overline{DX}$. Also, $\angle BXC$ and $\angle CXD$ are congruent right angles, and $\overline{CX} \cong \overline{CX}$. So, $\triangle BXC \cong \triangle DXC$ by the SAS Congruence Postulate. Corresponding parts of congruent triangles are congruent, so $\overline{BC} \cong \overline{DC}$. Opposite sides of a ▱*ABCD* are congruent, so $\overline{AD} \cong \overline{BC} \cong \overline{DC} \cong \overline{AB}$. By definition, *ABCD* is a rhombus.

EXAMPLE 4 Solve a real-world problem

CARPENTRY You are building a frame for a window. The window will be installed in the opening shown in the diagram.



a. The opening must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain*.

b. You measure the diagonals of the opening. The diagonals are 54.8 inches and 55.3 inches. What can you conclude about the shape of the opening?

Solution

a. No, you cannot. The boards on opposite sides are the same length, so they form a parallelogram. But you do not know whether the angles are right angles.

b. By Theorem 8.13, the diagonals of a rectangle are congruent. The diagonals of the quadrilateral formed by the boards are not congruent, so the boards do not form a rectangle.

✓ GUIDED PRACTICE for Example 4

4. Suppose you measure only the diagonals of a window opening. If the diagonals have the same measure, can you conclude that the opening is a rectangle? *Explain*.

8.4 EXERCISES

HOMEWORK KEY

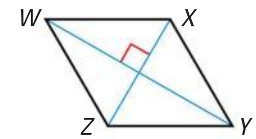
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 55

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 30, 31, and 62

SKILL PRACTICE

1. **VOCABULARY** What is another name for an equilateral rectangle?

2. ★ **WRITING** Do you have enough information to identify the figure at the right as a rhombus? *Explain.*



EXAMPLES 1, 2, and 3 on pp. 534–535 for Exs. 3–25

RHOMBUSES **For any rhombus *JKLM*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.**

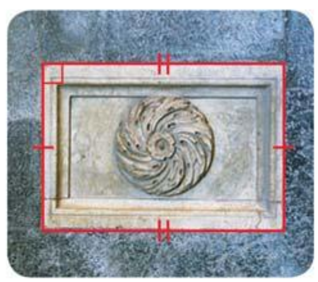
3. $\angle L \cong \angle M$
4. $\angle K \cong \angle M$
5. $\overline{JK} \cong \overline{KL}$
6. $\overline{JM} \cong \overline{KL}$
7. $\overline{JL} \cong \overline{KM}$
8. $\angle JKM \cong \angle LKM$

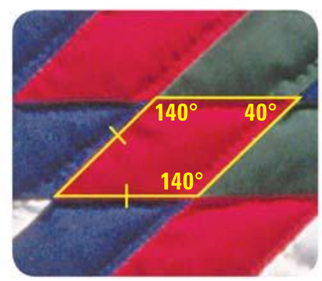
RECTANGLES **For any rectangle *WXYZ*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.**

9. $\angle W \cong \angle X$
10. $\overline{WX} \cong \overline{YZ}$
11. $\overline{WX} \cong \overline{XY}$
12. $\overline{WY} \cong \overline{XZ}$
13. $\overline{WY} \perp \overline{XZ}$
14. $\angle WXZ \cong \angle YXZ$

CLASSIFYING **Classify the quadrilateral. *Explain* your reasoning.**

15. 

16. 

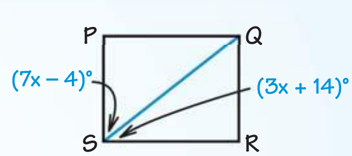
17. 

18. **USING PROPERTIES** Sketch rhombus *STUV*. *Describe* everything you know about the rhombus.

USING PROPERTIES **Name each quadrilateral—*parallelogram*, *rectangle*, *rhombus*, and *square*—for which the statement is true.**

19. It is equiangular.
20. It is equiangular and equilateral.
21. Its diagonals are perpendicular.
22. Opposite sides are congruent.
23. The diagonals bisect each other.
24. The diagonals bisect opposite angles.

25. **ERROR ANALYSIS** Quadrilateral *PQRS* is a rectangle. *Describe* and correct the error made in finding the value of *x*.



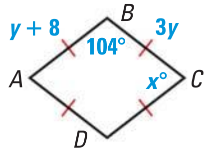
$7x - 4 = 3x + 14$

$4x = 18$

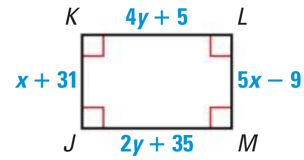
$x = 4.5$

xy ALGEBRA Classify the special quadrilateral. *Explain* your reasoning. Then find the values of *x* and *y*.

26.



27.



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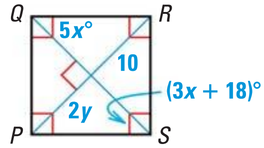

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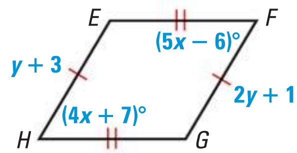
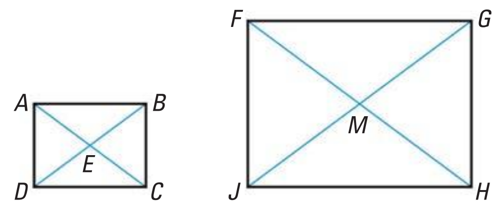


30. ★ **SHORT RESPONSE** The diagonals of a rhombus are 6 inches and 8 inches. What is the perimeter of the rhombus? *Explain.*

31. ★ **MULTIPLE CHOICE** Rectangle *ABCD* is similar to rectangle *FGHJ*. If $AC = 5$, $CD = 4$, and $FM = 5$, what is HJ?

Ⓐ 4 Ⓑ 5

Ⓒ 8 Ⓓ 10

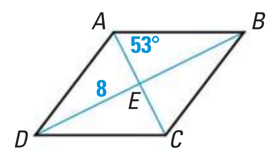


RHOMBUS The diagonals of rhombus *ABCD* intersect at *E*. Given that $m\angle BAC = 53°$ and $DE = 8$, find the indicated measure.

32. $m\angle DAC$ **33.** $m\angle AED$

34. $m\angle ADC$ **35.** DB

36. AE **37.** AC

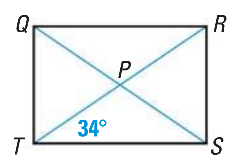


RECTANGLE The diagonals of rectangle *QRST* intersect at *P*. Given that $m\angle PTS = 34°$ and $QS = 10$, find the indicated measure.

38. $m\angle SRT$ **39.** $m\angle QPR$

40. QP **41.** RP

42. QR **43.** RS

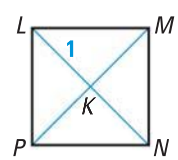


SQUARE The diagonals of square *LMNP* intersect at *K*. Given that $LK = 1$, find the indicated measure.

44. $m\angle MKN$ **45.** $m\angle LMK$

46. $m\angle LPK$ **47.** KN

48. MP **49.** LP



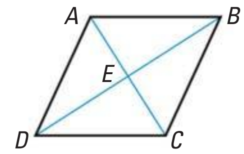
COORDINATE GEOMETRY Use the given vertices to graph ▱*JKLM*. Classify ▱*JKLM* and *explain* your reasoning. Then find the perimeter of ▱*JKLM*.

50. $J(-4, 2)$, $K(0, 3)$, $L(1, -1)$, $M(-3, -2)$

51. $J(-2, 7)$, $K(7, 2)$, $L(-2, -3)$, $M(-11, 2)$

52. REASONING Are all rhombuses similar? Are all squares similar? *Explain* your reasoning.

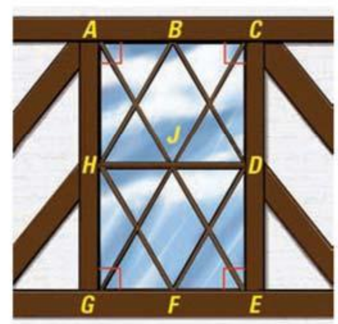
53. CHALLENGE Quadrilateral *ABCD* shown at the right is a rhombus. Given that $AC = 10$ and $BD = 16$, find all side lengths and angle measures. *Explain* your reasoning.



PROBLEM SOLVING

EXAMPLE 2 on p. 534 for Ex. 54

54. MULTI-STEP PROBLEM In the window shown at the right, $\overline{BD} \cong \overline{DF} \cong \overline{BH} \cong \overline{HF}$. Also, $\angle HAB$, $\angle BCD$, $\angle DEF$, and $\angle FGH$ are right angles.



a. Classify *HBDF* and *ACEG*. *Explain* your reasoning.

b. What can you conclude about the lengths of the diagonals $\overline{AE}$ and $\overline{GC}$? Given that these diagonals intersect at *J*, what can you conclude about the lengths of $\overline{AJ}$, $\overline{JE}$, $\overline{CJ}$, and $\overline{JG}$? *Explain*.

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EXAMPLE 4 on p. 536 for Ex. 55

55. PATIO You want to mark off a square region in your yard for a patio. You use a tape measure to mark off a quadrilateral on the ground. Each side of the quadrilateral is 2.5 meters long. *Explain* how you can use the tape measure to make sure that the quadrilateral you drew is a square.

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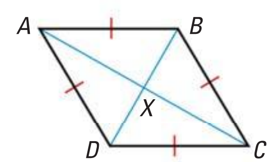
56. PROVING THEOREM 8.11 Use the plan for proof below to write a paragraph proof for the converse statement of Theorem 8.11.



GIVEN ▶ *ABCD* is a rhombus.

PROVE ▶ $\overline{AC} \perp \overline{BD}$

Plan for Proof Because *ABCD* is a parallelogram, its diagonals bisect each other at *X*. Show that $\triangle AXB \cong \triangle CXB$. Then show that $\overline{AC}$ and $\overline{BD}$ intersect to form congruent adjacent angles, $\angle AXB$ and $\angle CXB$.

PROVING COROLLARIES Write the corollary as a conditional statement and its converse. Then *explain* why each statement is true.

57. Rhombus Corollary

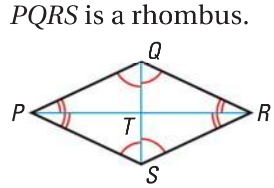
58. Rectangle Corollary

59. Square Corollary

PROVING THEOREM 8.12 In Exercises 60 and 61, prove both parts of Theorem 8.12.

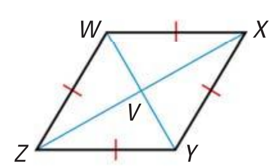
60. GIVEN ▶ *PQRS* is a parallelogram.
$\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$.
$\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.

PROVE ▶ *PQRS* is a rhombus.

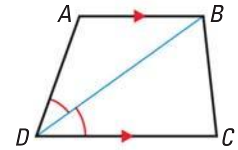


61. GIVEN ▶ *WXYZ* is a rhombus.

PROVE ▶ $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$.
$\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.



62. ★ **EXTENDED RESPONSE** In $ABCD$, $\overline{AB} \parallel \overline{CD}$, and $\overline{DB}$ bisects $\angle ADC$.



a. Show that $\angle ABD \cong \angle CDB$. What can you conclude about $\angle ADB$ and $\angle ABD$? What can you conclude about $\overline{AB}$ and $\overline{AD}$? *Explain*.

b. Suppose you also know that $\overline{AD} \cong \overline{BC}$. Classify $ABCD$. *Explain*.

63. **PROVING THEOREM 8.13** Write a coordinate proof of the following statement, which is part of Theorem 8.13.

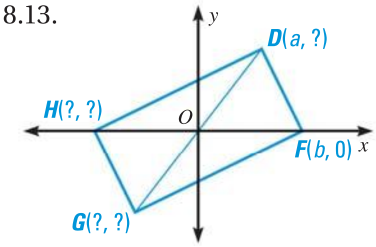
If a quadrilateral is a rectangle, then its diagonals are congruent.

64. **CHALLENGE** Write a coordinate proof of part of Theorem 8.13.

GIVEN ▶ $DFGH$ is a parallelogram, $\overline{DG} \cong \overline{HF}$

PROVE ▶ $DFGH$ is a rectangle.

Plan for Proof Write the coordinates of the vertices in terms of a and b. Find and compare the slopes of the sides.



MIXED REVIEW

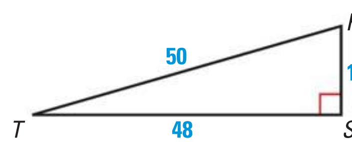
PREVIEW
Prepare for Lesson 8.5 in Ex. 65.

65. In $\triangle JKL$, $KL = 54.2$ centimeters. Point M is the midpoint of $\overline{JK}$ and N is the midpoint of $\overline{JL}$. Find MN. *(p. 295)*

Find the sine and cosine of the indicated angle. Write each answer as a fraction and a decimal. *(p. 473)*

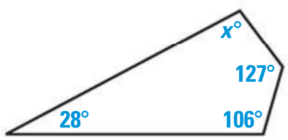
66. $\angle R$

67. $\angle T$

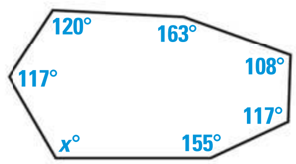


Find the value of x. *(p. 507)*

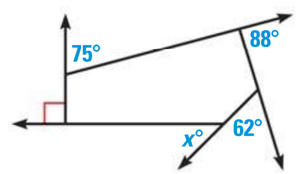
68.



69.



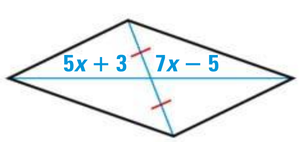
70.



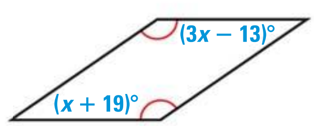
QUIZ *for Lessons 8.3–8.4*

For what value of x is the quadrilateral a parallelogram? *(p. 522)*

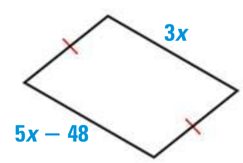
1.



2.

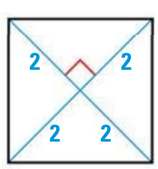


3.

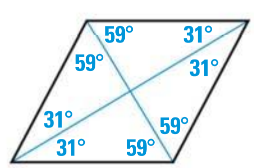


Classify the quadrilateral. *Explain* your reasoning. *(p. 533)*

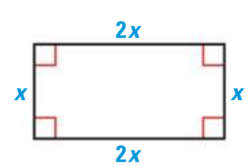
4.



5.



6.



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Keystrokes

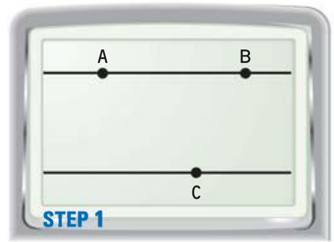
8.5 Midsegment of a Trapezoid

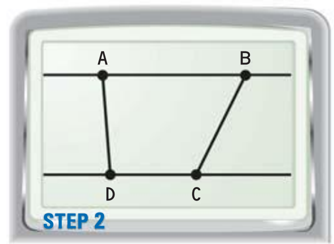
MATERIALS • graphing calculator or computer

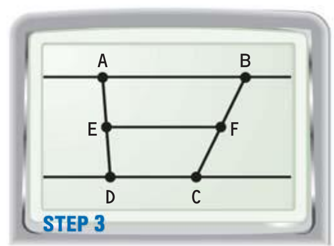
QUESTION What are the properties of the midsegment of a trapezoid?

You can use geometry drawing software to investigate properties of trapezoids.

EXPLORE Draw a trapezoid and its midsegment







STEP 1 ***Draw parallel lines*** Draw $\overleftrightarrow{AB}$. Draw a point C not on $\overleftrightarrow{AB}$ and construct a line parallel to $\overleftrightarrow{AB}$ through point C.

STEP 2 ***Draw trapezoid*** Construct a point D on the same line as point C. Then draw $\overline{AD}$ and $\overline{BC}$ so that the segments are not parallel. Draw $\overline{AB}$ and $\overline{DC}$. Quadrilateral $ABCD$ is called a *trapezoid*. A trapezoid is a quadrilateral with exactly one pair of parallel sides.

STEP 3 ***Draw midsegment*** Construct the midpoints of $\overline{AD}$ and $\overline{BC}$. Label the points E and F. Draw $\overline{EF}$. $\overline{EF}$ is called a *midsegment* of trapezoid $ABCD$. The midsegment of a trapezoid connects the midpoints of its nonparallel sides.

STEP 4 ***Measure lengths*** Measure AB, DC, and EF.

STEP 5 ***Compare lengths*** The average of AB and DC is $\frac{AB + DC}{2}$. Calculate and compare this average to EF. What do you notice? Drag point A or point B to change the shape of trapezoid $ABCD$. Do not allow $\overline{AD}$ to intersect $\overline{BC}$. What do you notice about EF and $\frac{AB + DC}{2}$?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about the length of the midsegment of a trapezoid.
2. The midsegment of a trapezoid is parallel to the two parallel sides of the trapezoid. What measurements could you make to show that the midsegment in the *Explore* is parallel to $\overline{AB}$ and $\overline{CD}$? *Explain.*
3. In Lesson 5.1 (page 295), you learned a theorem about the midsegment of a triangle. How is the midsegment of a trapezoid similar to the midsegment of a triangle? How is it different?

8.5 Use Properties of Trapezoids and Kites

Before You used properties of special parallelograms.

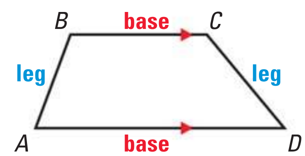
Now You will use properties of trapezoids and kites.

Why? So you can measure part of a building, as in Example 2.

Key Vocabulary
- **trapezoid** bases, base angles, legs
- **isosceles trapezoid**
- **midsegment of a trapezoid**
- **kite**

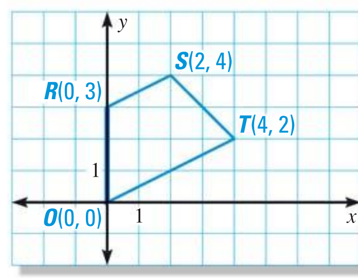
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

A trapezoid has two pairs of **base angles**. For example, in trapezoid *ABCD*, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the **legs** of the trapezoid.



EXAMPLE 1 Use a coordinate plane

Show that *ORST* is a trapezoid.



Solution

Compare the slopes of opposite sides.

Slope of $\overline{RS} = \frac{4-3}{2-0} = \frac{1}{2}$

Slope of $\overline{OT} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

The slopes of $\overline{RS}$ and $\overline{OT}$ are the same, so $\overline{RS} \parallel \overline{OT}$.

Slope of $\overline{ST} = \frac{2-4}{4-2} = \frac{-2}{2} = -1$

Slope of $\overline{OR} = \frac{3-0}{0-0} = \frac{3}{0}$, which is undefined.

The slopes of $\overline{ST}$ and $\overline{OR}$ are not the same, so $\overline{ST}$ is not parallel to $\overline{OR}$.

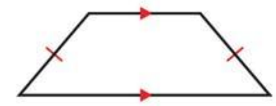
▶ Because quadrilateral *ORST* has exactly one pair of parallel sides, it is a trapezoid.

✓ GUIDED PRACTICE for Example 1

1. **WHAT IF?** In Example 1, suppose the coordinates of point *S* are (4, 5). What type of quadrilateral is *ORST*? *Explain*.

2. In Example 1, which of the interior angles of quadrilateral *ORST* are supplementary angles? *Explain* your reasoning.

ISOSCELES TRAPEZOIDS If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.



isosceles trapezoid

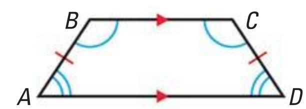
THEOREMS *For Your Notebook*

THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof: Ex. 37, p. 548

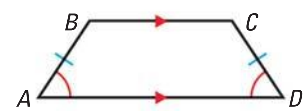


THEOREM 8.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

Proof: Ex. 38, p. 548

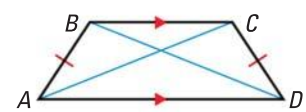


THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are congruent.

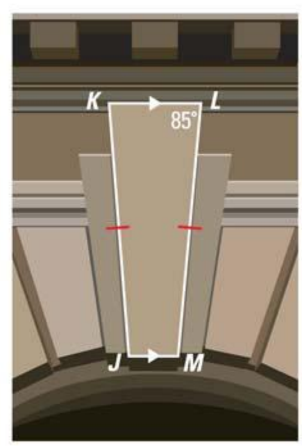
Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof: Exs. 39 and 43, p. 549



EXAMPLE 2 Use properties of isosceles trapezoids

ARCH The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$.



Solution

STEP 1 **Find $m\angle K$.** *JKLM* is an isosceles trapezoid, so $\angle K$ and $\angle L$ are congruent base angles, and $m\angle K = m\angle L = 85°$.

STEP 2 **Find $m\angle M$.** Because $\angle L$ and $\angle M$ are consecutive interior angles formed by $\overleftrightarrow{LM}$ intersecting two parallel lines, they are supplementary. So, $m\angle M = 180° - 85° = 95°$.

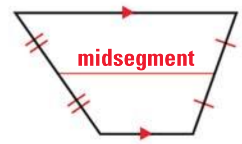
STEP 3 **Find $m\angle J$.** Because $\angle J$ and $\angle M$ are a pair of base angles, they are congruent, and $m\angle J = m\angle M = 95°$.

▶ So, $m\angle J = 95°$, $m\angle K = 85°$, and $m\angle M = 95°$.

READ VOCABULARY

The midsegment of a trapezoid is sometimes called the *median* of the trapezoid.

MIDSEGMENTS Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.

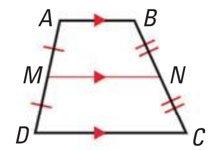


The theorem below is similar to the Midsegment Theorem for Triangles.

THEOREM *For Your Notebook*

THEOREM 8.17 Midsegment Theorem for Trapezoids

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

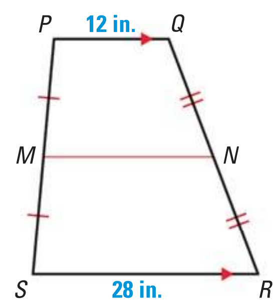


If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

Justification: Ex. 40, p. 549
Proof: p. 937

EXAMPLE 3 Use the midsegment of a trapezoid

In the diagram, $\overline{MN}$ is the midsegment of trapezoid $PQRS$. Find MN.



Solution

Use Theorem 8.17 to find MN.

$MN = \frac{1}{2}(PQ + SR)$ **Apply Theorem 8.17.**

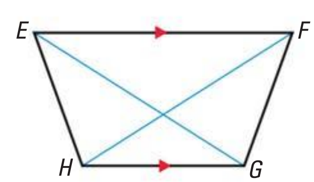
$= \frac{1}{2}(12 + 28)$ **Substitute 12 for *PQ* and 28 for *XU*.**

$= 20$ **Simplify.**

▶ The length MN is 20 inches.

✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, use the diagram of trapezoid *EFGH*.

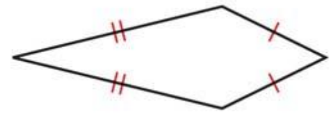


3. If $EG = FH$, is trapezoid $EFGH$ isosceles? *Explain.*

4. If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid $EFGH$ isosceles? *Explain.*

5. In trapezoid $JKLM$, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ cm. The length of the midsegment $\overline{NP}$ of trapezoid $JKLM$ is 12 cm. Sketch trapezoid $JKLM$ and its midsegment. Find ML. *Explain* your reasoning.

KITES A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



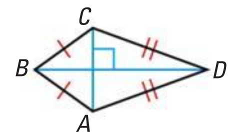
THEOREMS *For Your Notebook*

THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof: Ex. 41, p. 549

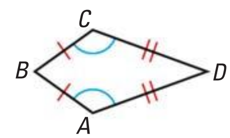


THEOREM 8.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof: Ex. 42, p. 549



EXAMPLE 4 Apply Theorem 8.19

Find $m\angle D$ in the kite shown at the right.

Solution

By Theorem 8.19, $DEFG$ has exactly one pair of congruent opposite angles. Because $\angle E \ncong \angle G$, $\angle D$ and $\angle F$ must be congruent. So, $m\angle D = m\angle F$. Write and solve an equation to find $m\angle D$.

$m\angle D + m\angle F + 124° + 80° = 360°$ **Corollary to Theorem 8.1**

$m\angle D + m\angle D + 124° + 80° = 360°$ **Substitute $m\angle D$ for $m\angle F$.**

$2(m\angle D) + 204° = 360°$ **Combine like terms.**

$m\angle D = 78°$ **Solve for $m\angle D$.**

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✓ GUIDED PRACTICE for Example 4

6. In a kite, the measures of the angles are $3x°$, 75°, 90°, and 120°. Find the value of x. What are the measures of the angles that are congruent?

8.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 19, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 28, 31, and 36

SKILL PRACTICE

1. **VOCABULARY** In trapezoid $PQRS$, $\overline{PQ} \parallel \overline{RS}$. Sketch $PQRS$ and identify its bases and its legs.

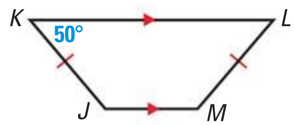
2. ★ **WRITING** *Describe* the differences between a kite and a trapezoid.

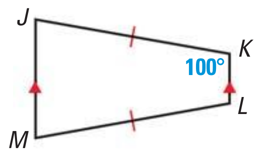
EXAMPLES 1 and 2 on pp. 542–543 for Exs. 3–12

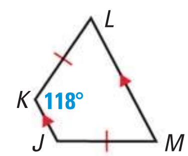
COORDINATE PLANE **Points *A*, *B*, *C*, and *D* are the vertices of a quadrilateral. Determine whether *ABCD* is a trapezoid.**

3. $A(0, 4)$, $B(4, 4)$, $C(8, -2)$, $D(2, 1)$
4. $A(-5, 0)$, $B(2, 3)$, $C(3, 1)$, $D(-2, -2)$
5. $A(2, 1)$, $B(6, 1)$, $C(3, -3)$, $D(-1, -4)$
6. $A(-3, 3)$, $B(-1, 1)$, $C(1, -4)$, $D(-3, 0)$

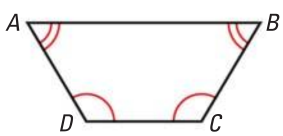
FINDING ANGLE MEASURES **Find $m\angle J$, $m\angle L$, and $m\angle M$.**

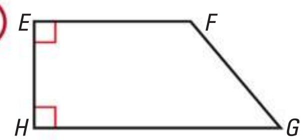
7. 

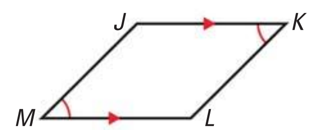
8. 

9. 

REASONING **Determine whether the quadrilateral is a trapezoid. *Explain*.**

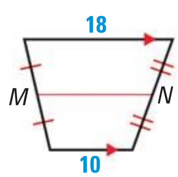
10. 

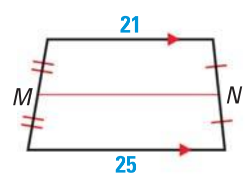
11. 

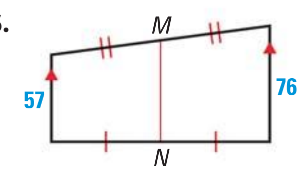
12. 

EXAMPLE 3 on p. 544 for Exs. 13–16

FINDING MIDSEGMENTS **Find the length of the midsegment of the trapezoid.**

13. 

14. 

15. 

16. ★ **MULTIPLE CHOICE** Which statement is not always true?

Ⓐ The base angles of an isosceles trapezoid are congruent.

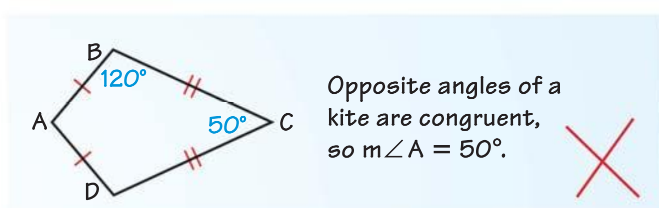
Ⓑ The midsegment of a trapezoid is parallel to the bases.

Ⓒ The bases of a trapezoid are parallel.

Ⓓ The legs of a trapezoid are congruent.

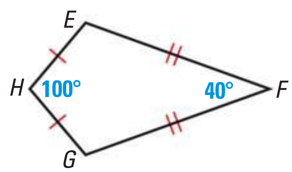
EXAMPLE 4 on p. 545 for Exs. 17–20

17. **ERROR ANALYSIS** *Describe* and correct the error made in finding $m\angle A$.

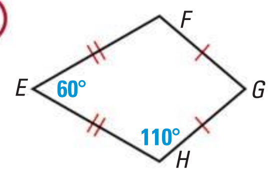


ANGLES OF KITES ***EFGH* is a kite. Find $m\angle G$.**

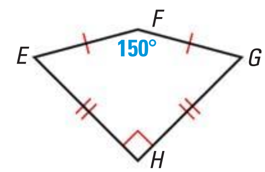
18.



19.

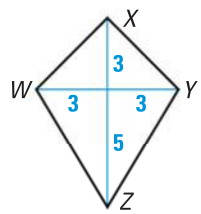


20.

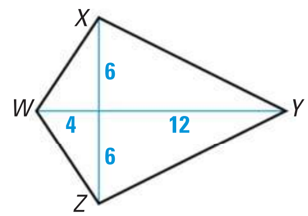


DIAGONALS OF KITES **Use Theorem 8.18 and the Pythagorean Theorem to find the side lengths of the kite. Write the lengths in simplest radical form.**

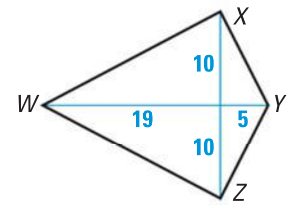
21.



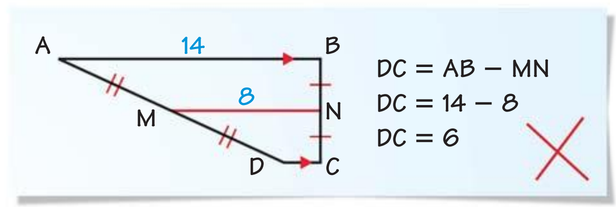
22.



23.

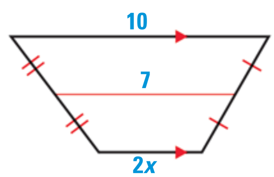


24. ERROR ANALYSIS In trapezoid *ABCD*, $\overline{MN}$ is the midsegment. *Describe* and correct the error made in finding *DC*.

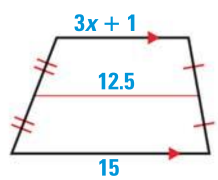


ALGEBRA **Find the value of x.**

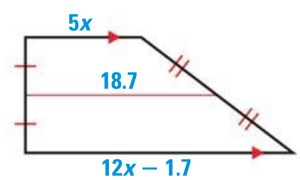
25.



26.



27.



28. ★ **SHORT RESPONSE** The points $M(-3, 5)$, $N(-1, 5)$, $P(3, -1)$, and $Q(-5, -1)$ form the vertices of a trapezoid. Draw *MNPQ* and find *MP* and *NQ*. What do your results tell you about the trapezoid? *Explain.*

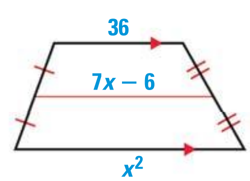
29. DRAWING In trapezoid *JKLM*, $\overline{JK} \parallel \overline{LM}$ and $JK = 17$. The midsegment of *JKLM* is $\overline{XY}$, and $XY = 37$. Sketch *JKLM* and its midsegment. Then find *LM*.

30. RATIOS The ratio of the lengths of the bases of a trapezoid is 1 : 3. The length of the midsegment is 24. Find the lengths of the bases.

31. ★ **MULTIPLE CHOICE** In trapezoid *PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{MN}$ is the midsegment of *PQRS*. If $RS = 5 \cdot PQ$, what is the ratio of *MN* to *RS*?

Ⓐ 3 : 5 Ⓑ 5 : 3 Ⓒ 2 : 1 Ⓓ 3 : 1

32. CHALLENGE The figure shown at the right is a trapezoid with its midsegment. Find all the possible values of x. What is the length of the midsegment? *Explain.* (The figure may not be drawn to scale.)

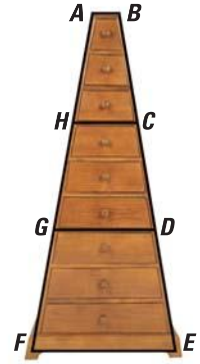


33. REASONING *Explain* why a kite and a general quadrilateral are the only quadrilaterals that can be concave.

PROBLEM SOLVING

EXAMPLES 3 and 4 on pp. 544–545 for Exs. 34–35

34. FURNITURE In the photograph of a chest of drawers, $\overline{HC}$ is the midsegment of trapezoid *ABDG*, $\overline{GD}$ is the midsegment of trapezoid *HCEF*, $AB = 13.9$ centimeters, and $GD = 50.5$ centimeters. Find *HC*. Then find *FE*.



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35. GRAPHIC DESIGN You design a logo in the shape of a convex kite. The measure of one angle of the kite is 90°. The measure of another angle is 30°. Sketch a kite that matches this description. Give the measures of all the angles and mark any congruent sides.

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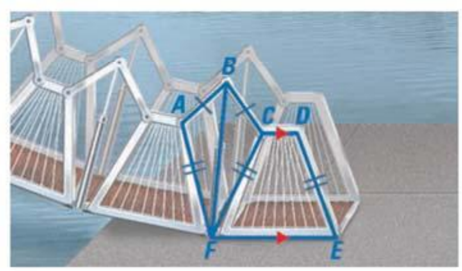
36. ★ **EXTENDED RESPONSE** The bridge below is designed to fold up into an octagon shape. The diagram shows a section of the bridge.

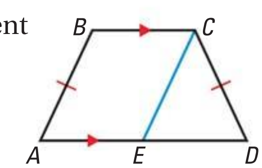
a. Classify the quadrilaterals shown in the diagram.

b. As the bridge folds up, what happens to the length of $\overline{BF}$? What happens to $m\angle BAF$, $m\angle ABC$, $m\angle BCF$, and $m\angle CFA$?

c. Given $m\angle CFE = 65°$, find $m\angle DEF$, $m\angle FCD$, and $m\angle CDE$. *Explain.*



37. PROVING THEOREM 8.14 Use the diagram and the auxiliary segment to prove Theorem 8.14. In the diagram, $\overline{EC}$ is drawn parallel to $\overline{AB}$.

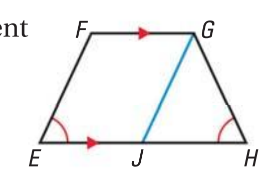


GIVEN ▶ *ABCD* is an isosceles trapezoid, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\angle A \cong \angle D$, $\angle B \cong \angle BCD$

Hint: Find a way to show that $\triangle ECD$ is an isosceles triangle.

38. PROVING THEOREM 8.15 Use the diagram and the auxiliary segment to prove Theorem 8.15. In the diagram, $\overline{JG}$ is drawn parallel to $\overline{EF}$.



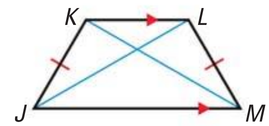
GIVEN ▶ *EFGH* is a trapezoid, $\overline{FG} \parallel \overline{EH}$, $\angle E \cong \angle H$

PROVE ▶ *EFGH* is an isosceles trapezoid.

Hint: Find a way to show that $\triangle JGH$ is an isosceles triangle.

◯ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

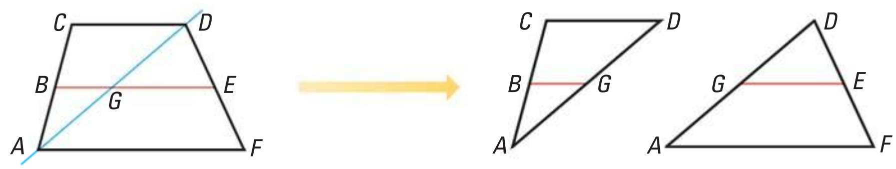
39. PROVING THEOREM 8.16 Prove part of Theorem 8.16.



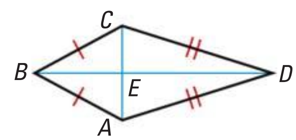
GIVEN ▶ *JKLM* is an isosceles trapezoid.
$\overline{KL} \parallel \overline{JM}$, $\overline{JK} \cong \overline{LM}$

PROVE ▶ $\overline{JL} \cong \overline{KM}$

40. REASONING In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$ and $\overline{GE}$ is the midsegment of $\triangle ADF$. *Explain* why the midsegment of trapezoid *ACDF* is parallel to each base and why its length is one half the sum of the lengths of the bases.



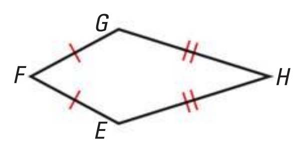
41. PROVING THEOREM 8.18 Prove Theorem 8.18.



GIVEN ▶ *ABCD* is a kite.
$\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

PROVE ▶ $\overline{AC} \perp \overline{BD}$

42. PROVING THEOREM 8.19 Write a paragraph proof of Theorem 8.19.



GIVEN ▶ *EFGH* is a kite.
$\overline{EF} \cong \overline{GF}$, $\overline{EH} \cong \overline{GH}$

PROVE ▶ $\angle E \cong \angle G$, $\angle F \ncong \angle H$

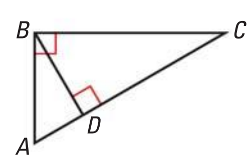
Plan for Proof First show that $\angle E \cong \angle G$. Then use an indirect argument to show that $\angle F \ncong \angle H$: If $\angle F \cong \angle H$, then *EFGH* is a parallelogram. But opposite sides of a parallelogram are congruent. This result contradicts the definition of a kite.

43. CHALLENGE In Exercise 39, you proved that part of Theorem 8.16 is true. Write the other part of Theorem 8.16 as a conditional statement. Then prove that the statement is true.

MIXED REVIEW

44. Place a right triangle in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex. *(p. 295)*

Use the diagram to complete the proportion. *(p. 449)*



45. $\dfrac{AB}{AC} = \dfrac{?}{AB}$

46. $\dfrac{AB}{BC} = \dfrac{BD}{?}$

PREVIEW
Prepare for Lesson 8.6 in Exs. 47–48.

Three of the vertices of $\square ABCD$ are given. Find the coordinates of point *D*. Show your method. *(p. 522)*

47. $A(-1, -2)$, $B(4, -2)$, $C(6, 2)$, $D(x, y)$

48. $A(1, 4)$, $B(0, 1)$, $C(4, 1)$, $D(x, y)$

Extension

Use after Lesson 8.5

Draw Three-Dimensional Figures

GOAL Create isometric drawings and orthographic projections of three-dimensional figures.

Key Vocabulary
- **isometric drawing**
- **orthographic projection**

Technical drawings are drawings that show different viewpoints of an object. Engineers and architects create technical drawings of products and buildings before actually constructing the actual objects.

EXAMPLE 1 Draw a rectangular box

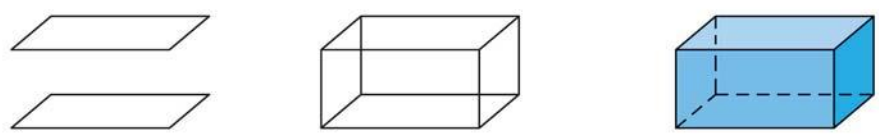
Draw a rectangular box.

Solution

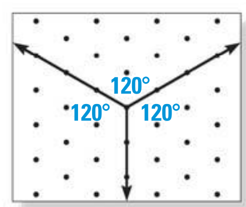
STEP 1 **Draw** the bases. They are rectangular, but you need to draw them tilted.

STEP 2 **Connect** the bases using vertical lines.

STEP 3 **Erase** parts of the hidden edges so that they are dashed lines.



ISOMETRIC DRAWINGS Technical drawings may include **isometric drawings**. These drawings look three-dimensional and can be created on a grid of dots using three axes that intersect to form 120° angles.



EXAMPLE 2 Create an isometric drawing

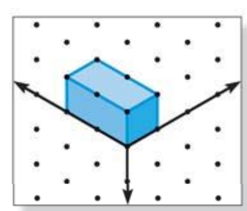
Create an isometric drawing of the rectangular box in Example 1.

Solution

STEP 1 **Draw** three axes on isometric dot paper.

STEP 2 **Draw** the box so that the edges of the box are parallel to the three axes.

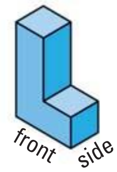
STEP 3 **Add** depth to the drawing by using different shading for the front, top, and sides.



ANOTHER VIEW Technical drawings may also include an *orthographic projection*. An **orthographic projection** is a two-dimensional drawing of the front, top, and side views of an object. The interior lines in these two-dimensional drawings represent edges of the object.

EXAMPLE 3 Create an orthographic projection

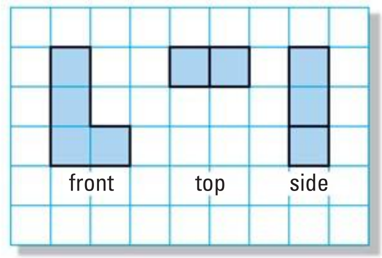
Create an orthographic projection of the solid.



VISUAL REASONING
In this Extension, you can think of the solids as being constructed from cubes. You can assume there are no cubes hidden from view except those needed to support the visible ones.

Solution

On graph paper, draw the front, top, and side views of the solid.



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PRACTICE

EXAMPLE 1 on p. 550 for Exs. 1–3

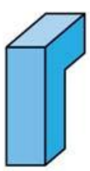
DRAWING BOXES **Draw a box with the indicated base.**

1. Equilateral triangle **2.** Regular hexagon **3.** Square

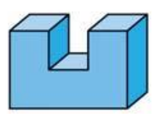
EXAMPLES 2 and 3 on pp. 550–551 for Exs. 4–12

DRAWING SOLIDS **Create an isometric drawing of the solid. Then create an orthographic projection of the solid.**

4.



5.



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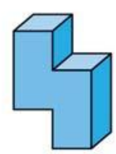

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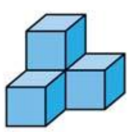

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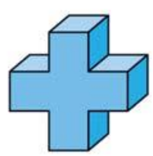

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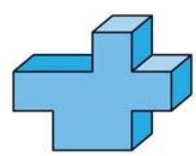
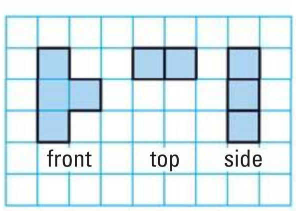
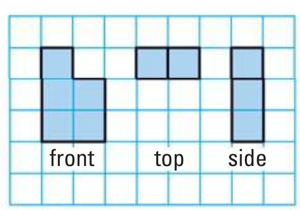


CREATING ISOMETRIC DRAWINGS **Create an isometric drawing of the orthographic projection.**

10.



11.



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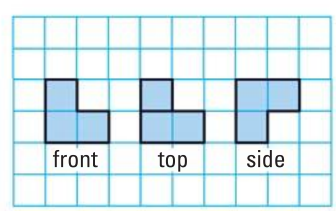


8.6 Identify Special Quadrilaterals

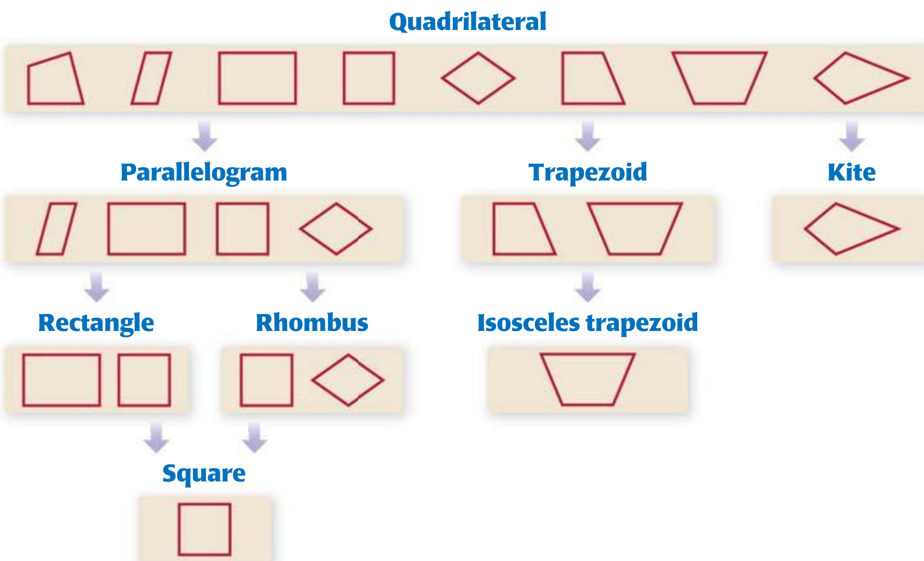
Before You identified polygons.

Now You will identify special quadrilaterals.

Why? So you can describe part of a pyramid, as in Ex. 36.

Key Vocabulary
- **parallelogram,** *p. 515*
- **rhombus,** *p. 533*
- **rectangle,** *p. 533*
- **square,** *p. 533*
- **trapezoid,** *p. 542*
- **kite,** *p. 545*

The diagram below shows relationships among the special quadrilaterals you have studied in Chapter 8. Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.

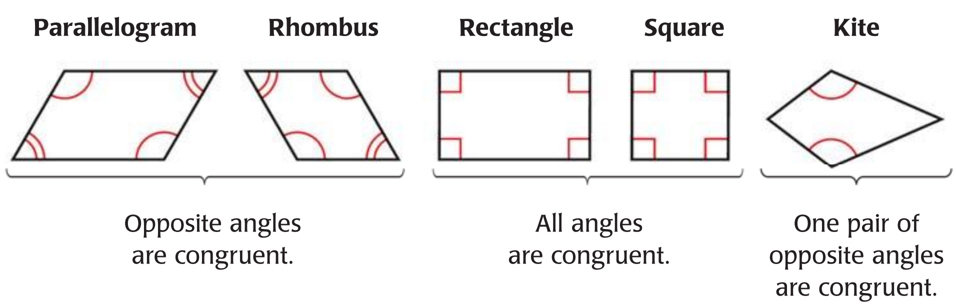


EXAMPLE 1 Identify quadrilaterals

Quadrilateral *ABCD* has at least one pair of opposite angles congruent. What types of quadrilaterals meet this condition?

Solution

There are many possibilities.



EXAMPLE 2 Standardized Test Practice

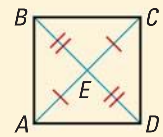
AVOID ERRORS

In Example 2, *ABCD* is shaped like a square. But you must rely only on marked information when you interpret a diagram.

What is the most specific name for quadrilateral *ABCD*?

Ⓐ Parallelogram   Ⓑ Rhombus

Ⓒ Square   Ⓓ Rectangle



Solution

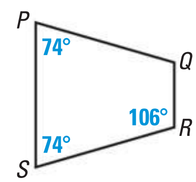
The diagram shows $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. So, the diagonals bisect each other. By Theorem 8.10, *ABCD* is a parallelogram.

Rectangles, rhombuses and squares are also parallelograms. However, there is no information given about the side lengths or angle measures of *ABCD*. So, you cannot determine whether it is a rectangle, a rhombus, or a square.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 3 Identify a quadrilateral

Is enough information given in the diagram to show that quadrilateral *PQRS* is an isosceles trapezoid? Explain.



Solution

STEP 1 **Show** that *PQRS* is a trapezoid. $\angle R$ and $\angle S$ are supplementary, but $\angle P$ and $\angle S$ are not. So, $\overline{PS} \parallel \overline{QR}$, but $\overline{PQ}$ is not parallel to $\overline{SR}$. By definition, *PQRS* is a trapezoid.

STEP 2 **Show** that trapezoid *PQRS* is isosceles. $\angle P$ and $\angle S$ are a pair of congruent base angles. So, *PQRS* is an isosceles trapezoid by Theorem 8.15.

▶ Yes, the diagram is sufficient to show that *PQRS* is an isosceles trapezoid.

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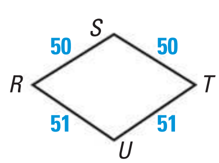
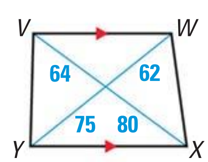
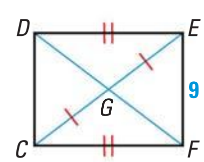
✓ GUIDED PRACTICE for Examples 1, 2, and 3

1. Quadrilateral *DEFG* has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

Give the most specific name for the quadrilateral. *Explain* your reasoning.

2. 

3. 

4. 

5. **ERROR ANALYSIS** A student knows the following information about quadrilateral *MNPQ*: $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \cong \overline{NQ}$, and $\angle P \cong \angle Q$. The student concludes that *MNPQ* is an isosceles trapezoid. *Explain* why the student cannot make this conclusion.

8.6 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 15, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 37, and 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A quadrilateral that has exactly one pair of parallel sides and diagonals that are congruent is a(n) __?__.

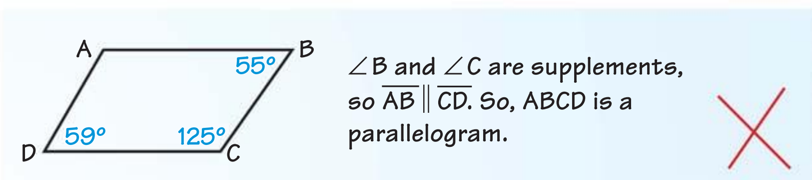
2. ★ **WRITING** *Describe* three methods you could use to prove that a parallelogram is a rhombus.

EXAMPLE 1 on p. 552 for Exs. 3–12

PROPERTIES OF QUADRILATERALS **Copy the chart. Put an X in the box if the shape *always* has the given property.**

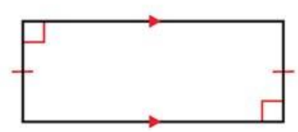
	Property	▱	Rectangle	Rhombus	Square	Kite	Trapezoid
3.	All sides are ≅.	?	?	?	?	?	?
4.	Both pairs of opp. sides are ≅.	?	?	?	?	?	?
5.	Both pairs of opp. sides are ∥.	?	?	?	?	?	?
6.	Exactly 1 pair of opp. sides are ∥.	?	?	?	?	?	?
7.	All ∠s are ≅.	?	?	?	?	?	?
8.	Exactly 1 pair of opp. ∠s are ≅.	?	?	?	?	?	?
9.	Diagonals are ⊥.	?	?	?	?	?	?
10.	Diagonals are ≅.	?	?	?	?	?	?
11.	Diagonals bisect each other.	?	?	?	?	?	?

12. **ERROR ANALYSIS** *Describe* and correct the error in classifying the quadrilateral.



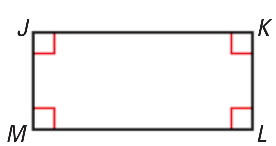
EXAMPLE 2 on p. 553 for Exs. 13–17

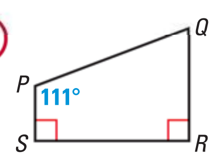
13. ★ **MULTIPLE CHOICE** What is the most specific name for the quadrilateral shown at the right?

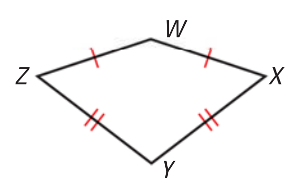


Ⓐ Rectangle Ⓑ Parallelogram
Ⓒ Trapezoid Ⓓ Isosceles trapezoid

CLASSIFYING QUADRILATERALS **Give the most specific name for the quadrilateral. *Explain*.**

14. 

15. 

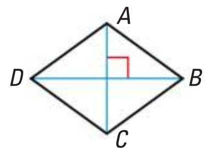
16. 

17. **DRAWING** Draw a quadrilateral with congruent diagonals and exactly one pair of congruent sides. What is the most specific name for this quadrilateral?

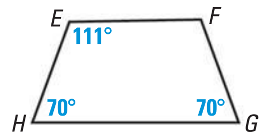
EXAMPLE 3
on p. 553
for Exs. 18–20

IDENTIFYING QUADRILATERALS Tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. *Explain*.

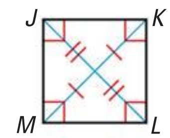
18. Rhombus



19. Isosceles trapezoid



20. Square



COORDINATE PLANE Points *P*, *Q*, *R*, and *S* are the vertices of a quadrilateral. Give the most specific name for *PQRS*. *Justify* your answer.

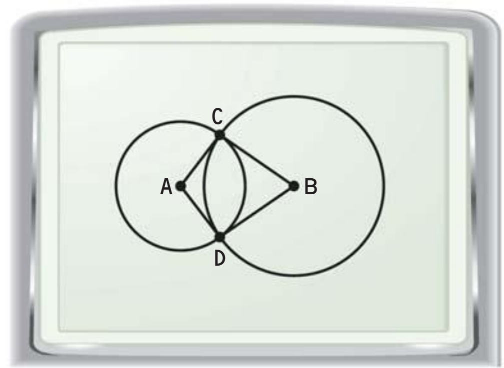
21. $P(1, 0)$, $Q(1, 2)$, $R(6, 5)$, $S(3, 0)$

22. $P(2, 1)$, $Q(6, 1)$, $R(5, 8)$, $S(3, 8)$

23. $P(2, 7)$, $Q(6, 9)$, $R(9, 3)$, $S(5, 1)$

24. $P(1, 7)$, $Q(5, 8)$, $R(6, 2)$, $S(2, 1)$

25. **TECHNOLOGY** Use geometry drawing software to draw points *A*, *B*, *C*, and segments *AC* and *BC*. Draw a circle with center *A* and radius *AC*. Draw a circle with center *B* and radius *BC*. Label the other intersection of the circles *D*. Draw $\overline{BD}$ and $\overline{AD}$.

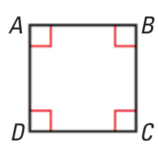


   a. Drag point *A*, *B*, *C*, or *D* to change the shape of *ABCD*. What types of quadrilaterals can be formed?

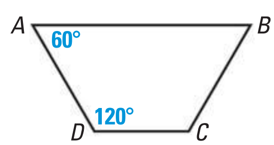
   b. Are there types of quadrilaterals that cannot be formed? *Explain*.

DEVELOPING PROOF Which pairs of segments or angles must be congruent so that you can prove that *ABCD* is the indicated quadrilateral? *Explain*. There may be more than one right answer.

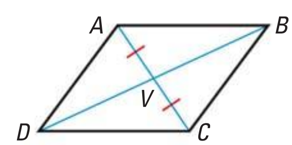
26. Square



27. Isosceles trapezoid



28. Parallelogram

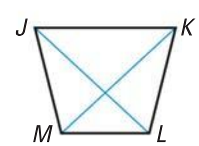


TRAPEZOIDS In Exercises 29–31, determine whether there is enough information to prove that *JKLM* is an isosceles trapezoid. *Explain*.

29. **GIVEN** ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JKL \cong \angle KJM$

30. **GIVEN** ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JML \cong \angle KLM$, $m\angle KLM \neq 90°$

31. **GIVEN** ▶ $\overline{JL} \cong \overline{KM}$, $\overline{JK} \parallel \overline{LM}$, $JK > LM$



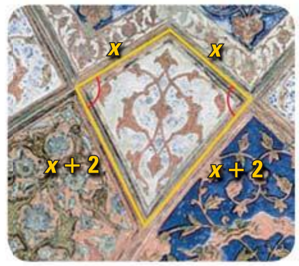
32. **CHALLENGE** Draw a rectangle and bisect its angles. What type of quadrilateral is formed by the intersecting bisectors? *Justify* your answer.

PROBLEM SOLVING

REAL-WORLD OBJECTS **What type of special quadrilateral is outlined?**

33.

34.



35.

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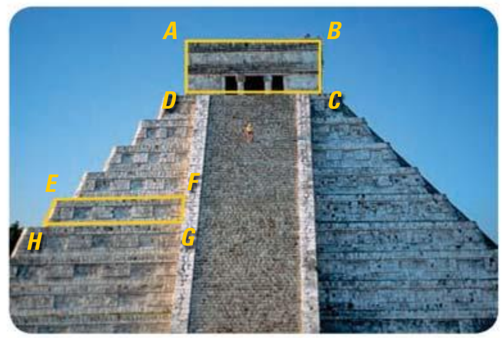
36. PYRAMID Use the photo of the Pyramid of Kukulcan in Mexico.



a. $\overline{EF} \parallel \overline{HG}$, and $\overline{EH}$ and $\overline{FG}$ are not parallel. What shape is this part of the pyramid?

b. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$, and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are all congruent to each other. What shape is this part of the pyramid?

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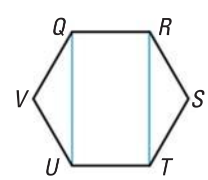
37. ★ **SHORT RESPONSE** *Explain* why a parallelogram with one right angle must be a rectangle.

38. ★ **EXTENDED RESPONSE** Segments *AC* and *BD* bisect each other.

a. Suppose that $\overline{AC}$ and $\overline{BD}$ are congruent, but not perpendicular. Draw quadrilateral *ABCD* and classify it. *Justify* your answer.

b. Suppose that $\overline{AC}$ and $\overline{BD}$ are perpendicular, but not congruent. Draw quadrilateral *ABCD* and classify it. *Justify* your answer.

39. MULTI-STEP PROBLEM Polygon *QRSTUV* shown at the right is a regular hexagon, and $\overline{QU}$ and $\overline{RT}$ are diagonals. Follow the steps below to classify quadrilateral *QRTU*. *Explain* your reasoning in each step.



a. Show that $\triangle QVU$ and $\triangle RST$ are congruent isosceles triangles.

b. Show that $\overline{QR} \cong \overline{UT}$ and that $\overline{QU} \cong \overline{RT}$.

c. Show that $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$. Find the measure of each of these angles.

d. Classify quadrilateral *QRTU*.

40. REASONING In quadrilateral *WXYZ*, $\overline{WY}$ and $\overline{XZ}$ intersect each other at point *V*. $\overline{WV} \cong \overline{XV}$ and $\overline{YV} \cong \overline{ZV}$, but $\overline{WY}$ and $\overline{XZ}$ do not bisect each other. Draw $\overline{WY}$, $\overline{XY}$, and *WXYZ*. What special type of quadrilateral is *WXYZ*? Write a plan for a proof of your answer.

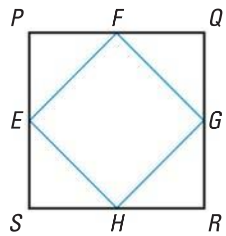
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

CHALLENGE **What special type of quadrilateral is *EFGH*? Write a paragraph proof to show that your answer is correct.**

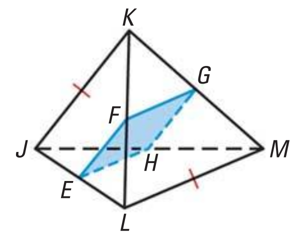
41. **GIVEN ▶** *PQRS* is a square. *E*, *F*, *G*, and *H* are midpoints of the sides of the square.

PROVE ▶ *EFGH* is a __?__.



42. **GIVEN ▶** In the three-dimensional figure, $\overline{JK} \cong \overline{LM}$; *E*, *F*, *G*, and *H* are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$.

PROVE ▶ *EFGH* is a __?__.

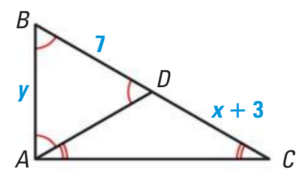


MIXED REVIEW

In Exercises 43 and 44, use the diagram. *(p. 264)*

43. Find the values of *x* and *y*. *Explain* your reasoning.

44. Find $m\angle ADC$, $m\angle DAC$, and $m\angle DCA$. *Explain* your reasoning.



PREVIEW
Prepare for Lesson 9.1 in Exs. 45–46.

The vertices of quadrilateral *ABCD* are *A*(−2, 1), *B*(2, 5), *C*(3, 2), and *D*(1, −1). Draw *ABCD* in a coordinate plane. Then draw its image after the indicated translation. *(p. 272)*

45. $(x, y) \rightarrow (x + 1, y - 3)$

46. $(x, y) \rightarrow (x - 2, y - 2)$

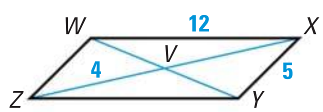
Use the diagram of ▱*WXYZ* to find the indicated length. *(p. 515)*

47. *YZ*

48. *WZ*

49. *XV*

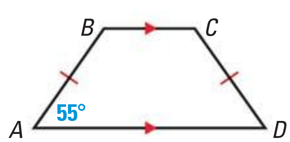
50. *XZ*



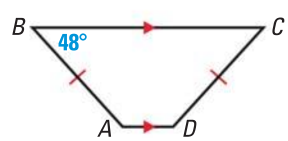
QUIZ for Lessons 8.5–8.6

Find the unknown angle measures. *(p. 542)*

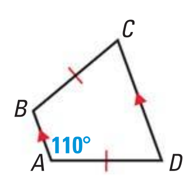
1.



2.



3.



4. The diagonals of quadrilateral *ABCD* are congruent and bisect each other. What types of quadrilaterals match this description? *(p. 552)*

5. In quadrilateral *EFGH*, $\angle E \cong \angle G$, $\angle F \cong \angle H$, and $\overline{EF} \cong \overline{EH}$. What is the most specific name for quadrilateral *EFGH*? *(p. 552)*

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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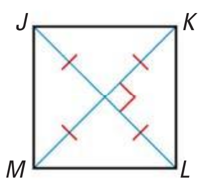
Lessons 8.4–8.6

1. **MULTI-STEP PROBLEM** In the photograph shown below, quadrilateral *ABCD* represents the front view of the roof.

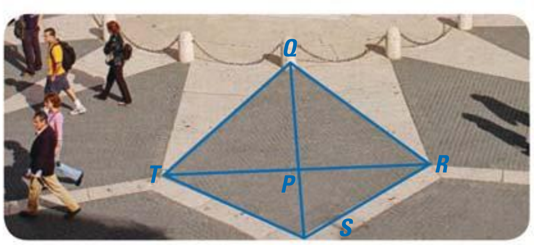
   a. *Explain* how you know that the shape of the roof is a trapezoid.

   b. Do you have enough information to determine that the roof is an isosceles trapezoid? *Explain* your reasoning.

2. **SHORT RESPONSE** Is enough information given in the diagram to show that quadrilateral *JKLM* is a square? *Explain* your reasoning.



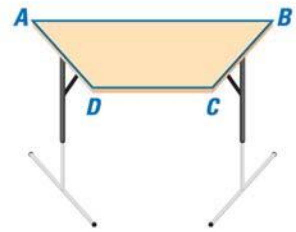
3. **EXTENDED RESPONSE** In the photograph, quadrilateral *QRST* is a kite.



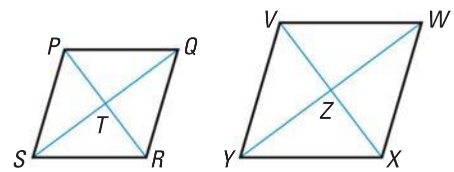
   a. If $m\angle TQR = 102°$ and $m\angle RST = 125°$, find $m\angle QTS$. *Explain* your reasoning.

   b. If $QS = 11$ ft, $TR = 14$ ft, and $\overline{TP} \cong \overline{QP} \cong \overline{RP}$, find *QR*, *RS*, *ST*, and *TQ*. Round your answers to the nearest foot. Show your work.

4. **GRIDDED ANSWER** The top of the table shown is shaped like an isosceles trapezoid. In *ABCD*, $AB = 48$ inches, $BC = 19$ inches, $CD = 24$ inches, and $DA = 19$ inches. Find the length (in inches) of the midsegment of *ABCD*.



5. **SHORT RESPONSE** Rhombus *PQRS* is similar to rhombus *VWXY*. In the diagram below, $QS = 32$, $QR = 20$, and $WZ = 20$. Find *WX*. *Explain* your reasoning.



6. **OPEN-ENDED** In quadrilateral *MNPQ*, $\overline{MP} \cong \overline{NQ}$.

   a. What types of quadrilaterals could *MNPQ* be? Use the most specific names. *Explain*.

   b. For each of your answers in part (a), tell what additional information would allow you to conclude that *MNPQ* is that type of quadrilateral. *Explain* your reasoning. (There may be more than one correct answer.)

7. **EXTENDED RESPONSE** Three of the vertices of quadrilateral *EFGH* are $E(0, 4)$, $F(2, 2)$, and $G(4, 4)$.

   a. Suppose that *EFGH* is a rhombus. Find the coordinates of vertex *H*. *Explain* why there is only one possible location for *H*.

   b. Suppose that *EFGH* is a convex kite. Show that there is more than one possible set of coordinates for vertex *H*. *Describe* what all the possible sets of coordinates have in common.

8 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Using Angle Relationships in Polygons

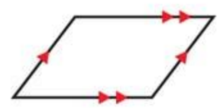
You can use theorems about the interior and exterior angles of convex polygons to solve problems.

Polygon Interior Angles Theorem	Polygon Exterior Angles Theorem
The sum of the interior angle measures of a convex n-gon is $(n - 2) \cdot 180°$.	The sum of the exterior angle measures of a convex n-gon is $360°$.

Big Idea 2

Using Properties of Parallelograms

By definition, a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Other properties of parallelograms:



- Opposite sides are congruent.
- Diagonals bisect each other.
- Opposite angles are congruent.
- Consecutive angles are supplementary.

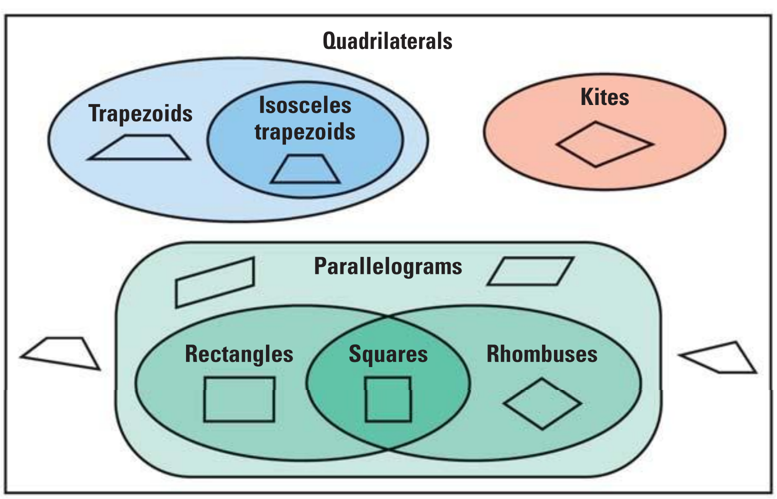
Ways to show that a quadrilateral is a parallelogram:

- Show both pairs of opposite sides are parallel.
- Show both pairs of opposite sides or opposite angles are congruent.
- Show one pair of opposite sides are congruent and parallel.
- Show the diagonals bisect each other.

Big Idea 3

Classifying Quadrilaterals by Their Properties

Special quadrilaterals can be classified by their properties. In a parallelogram, both pairs of opposite sides are parallel. In a trapezoid, only one pair of sides are parallel. A kite has two pairs of consecutive congruent sides, but opposite sides are not congruent.



8 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- diagonal, *p. 507*
- parallelogram, *p. 515*
- rhombus, *p. 533*
- rectangle, *p. 533*
- square, *p. 533*
- trapezoid, *p. 542*
- bases of a trapezoid, *p. 542*
- base angles of a trapezoid, *p. 542*
- legs of a trapezoid, *p. 542*
- isosceles trapezoid, *p. 543*
- midsegment of a trapezoid, *p. 544*
- kite, *p. 545*

VOCABULARY EXERCISES

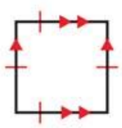
In Exercises 1 and 2, copy and complete the statement.

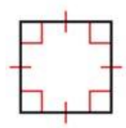
1. The _?_ of a trapezoid is parallel to the bases.

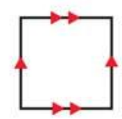
2. A(n) _?_ of a polygon is a segment whose endpoints are nonconsecutive vertices.

3. **WRITING** *Describe* the different ways you can show that a trapezoid is an isosceles trapezoid.

In Exercises 4–6, match the figure with the most specific name.

4. 

5. 

6. 

A. Square **B.** Parallelogram **C.** Rhombus

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 8.

8.1 Find Angle Measures in Polygons *pp. 507–513*

EXAMPLE

The sum of the measures of the interior angles of a convex regular polygon is 1080°. Classify the polygon by the number of sides. What is the measure of each interior angle?

Write and solve an equation for the number of sides n.

$(n - 2) \cdot 180° = 1080°$ **Polygon Interior Angles Theorem**

$n = 8$ **Solve for *n*.**

The polygon has 8 sides, so it is an octagon.

A regular octagon has 8 congruent interior angles, so divide to find the measure of each angle: $1080° \div 8 = 135°$. The measure of each interior angle is 135°.

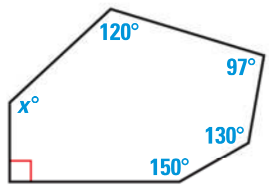
EXERCISES

EXAMPLES 2, 3, 4, and 5 on pp. 508–510 for Exs. 7–11

7. The sum of the measures of the interior angles of a convex regular polygon is 3960°. Classify the polygon by the number of sides. What is the measure of each interior angle?

In Exercises 8–10, find the value of *x*.

8.



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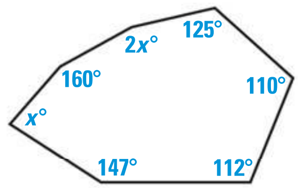
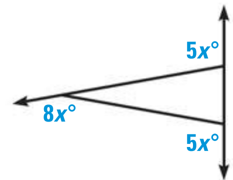


10.



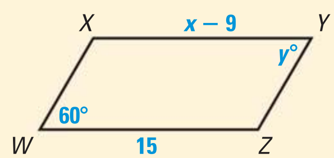
11. In a regular nonagon, the exterior angles are all congruent. What is the measure of one of the exterior angles? *Explain.*

8.2 Use Properties of Parallelograms

pp. 515–521

EXAMPLE

Quadrilateral *WXYZ* is a parallelogram. Find the values of *x* and *y*.



To find the value of x, apply Theorem 8.3.

$XY = WZ$	**Opposite sides of a ▱ are ≅.**
$x - 9 = 15$	**Substitute.**
$x = 24$	**Add 9 to each side.**

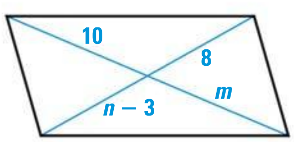
By Theorem 8.4, $\angle W \cong \angle Y$, or $m\angle W = m\angle Y$. So, $y = 60$.

EXERCISES

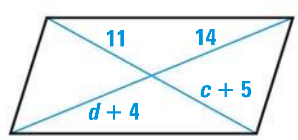
EXAMPLES 1, 2, and 3 on pp. 515, 517 for Exs. 12–17

Find the value of each variable in the parallelogram.

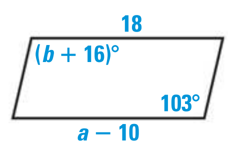
12.



13.



14.



15. In ▱$PQRS$, $PQ = 5$ centimeters, $QR = 10$ centimeters, and $m\angle PQR = 36°$. Sketch $PQRS$. Find and label all of its side lengths and interior angle measures.

16. The perimeter of ▱$EFGH$ is 16 inches. If EF is 5 inches, find the lengths of all the other sides of $EFGH$. *Explain* your reasoning.

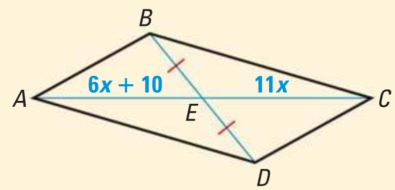
17. In ▱$JKLM$, the ratio of the measure of $\angle J$ to the measure of $\angle M$ is 5 : 4. Find $m\angle J$ and $m\angle M$. *Explain* your reasoning.

8 CHAPTER REVIEW

8.3 Show that a Quadrilateral is a Parallelogram *pp. 522–529*

EXAMPLE

For what value of *x* is quadrilateral *ABCD* a parallelogram?



If the diagonals bisect each other, then *ABCD* is a parallelogram. The diagram shows that $\overline{BE} \cong \overline{DE}$. You need to find the value of *x* that makes $\overline{AE} \cong \overline{CE}$.

$AE = CE$ — **Set the segment lengths equal.**

$6x + 10 = 11x$ — **Substitute expressions for the lengths.**

$x = 2$ — **Solve for x.**

When $x = 2$, $AE = 6(2) + 10 = 22$ and $CE = 11(2) = 22$. So, $\overline{AE} \cong \overline{CE}$.

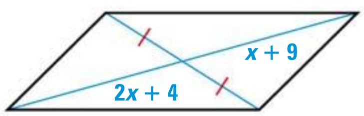
Quadrilateral *ABCD* is a parallelogram when $x = 2$.

EXERCISES

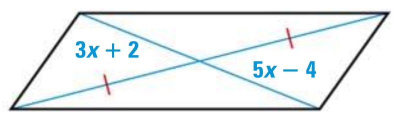
EXAMPLE 3 on p. 524 for Exs. 18–19

For what value of *x* is the quadrilateral a parallelogram?

18.



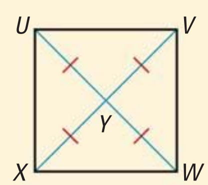
19.



8.4 Properties of Rhombuses, Rectangles, and Squares *pp. 533–540*

EXAMPLE

Classify the special quadrilateral.



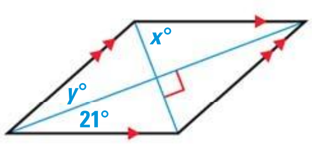
In quadrilateral *UVWX*, the diagonals bisect each other. So, *UVWX* is a parallelogram. Also, $\overline{UY} \cong \overline{VY} \cong \overline{WY} \cong \overline{XY}$. So, $UY + YW = VY + XY$. Because $UY + YW = UW$, and $VY + XY = VX$, you can conclude that $\overline{UW} \cong \overline{VX}$. By Theorem 8.13, *UVWX* is a rectangle.

EXERCISES

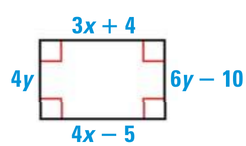
EXAMPLES 2 and 3 on pp. 534–535 for Exs. 20–22

Classify the special quadrilateral. Then find the values of *x* and *y*.

20.



21.



22. The diagonals of a rhombus are 10 centimeters and 24 centimeters. Find the length of a side. *Explain.*

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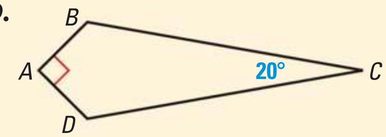
Chapter Review Practice

8.5 Use Properties of Trapezoids and Kites

pp. 542–549

EXAMPLE

Quadrilateral *ABCD* is a kite. Find $m\angle B$ and $m\angle D$.



A kite has exactly one pair of congruent opposite angles. Because $\angle A \ncong \angle C$, $\angle B$ and $\angle D$ must be congruent. Write and solve an equation.

$90° + 20° + m\angle B + m\angle D = 360°$ **Corollary to Theorem 8.1**

$110° + m\angle B° + m\angle D = 360°$ **Combine like terms.**

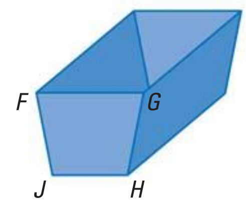
$m\angle B + m\angle D = 250°$ **Subtract 110° from each side.**

Because $\angle B \cong \angle D$, you can substitute $m\angle B$ for $m\angle D$ in the last equation. Then $m\angle B + m\angle B = 250°$, and $m\angle B = m\angle D = 125°$.

EXERCISES

EXAMPLES 2 and 3 on pp. 543–544 for Exs. 20–22

In Exercises 23 and 24, use the diagram of a recycling container. One end of the container is an isosceles trapezoid with $\overline{FG} \parallel \overline{JH}$ and $m\angle F = 79°$.



23. Find $m\angle G$, $m\angle H$, and $m\angle J$.

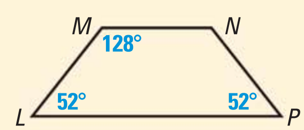
24. Copy trapezoid *FGHJ* and sketch its midsegment. If the midsegment is 16.5 inches long and $\overline{FG}$ is 19 inches long, find *JH*.

8.6 Identify Special Quadrilaterals

pp. 552–557

EXAMPLE

Give the most specific name for quadrilateral *LMNP*.



In *LMNP*, $\angle L$ and $\angle M$ are supplementary, but $\angle L$ and $\angle P$ are not. So, $\overline{MN} \parallel \overline{LP}$, but $\overline{LM}$ is not parallel to $\overline{NP}$. By definition, *LMNP* is a trapezoid.

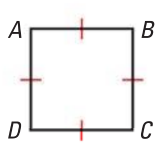
Also, $\angle L$ and $\angle P$ are a pair of base angles and $\angle L \cong \angle P$. So, *LMNP* is an isosceles trapezoid by Theorem 8.15.

EXERCISES

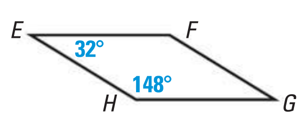
EXAMPLE 2 on p. 553 for Exs. 25–28

Give the most specific name for the quadrilateral. *Explain* your reasoning.

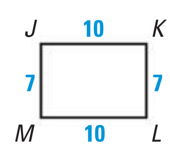
25.



26.



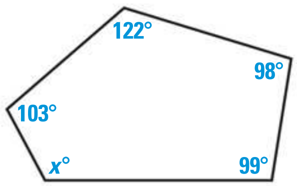
27.

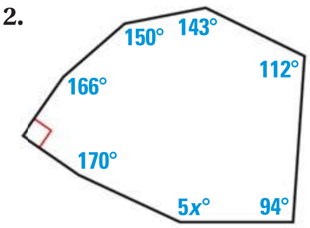


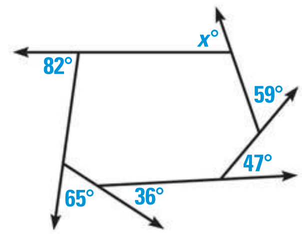
28. In quadrilateral *RSTU*, $\angle R$, $\angle T$, and $\angle U$ are right angles, and $RS = ST$. What is the most specific name for quadrilateral *RSTU*? *Explain.*

8 CHAPTER TEST

Find the value of *x*.

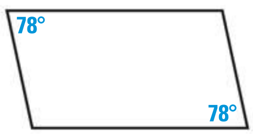
1.


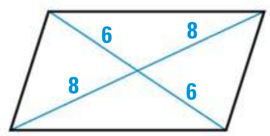
2.


3.


4. In ▱*EFGH*, $m\angle F$ is 40° greater than $m\angle G$. Sketch ▱*EFGH* and label each angle with its correct angle measure. *Explain* your reasoning.

Are you given enough information to determine whether the quadrilateral is a parallelogram? *Explain* your reasoning.

5.


6.


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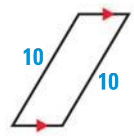


In Exercises 8–11, list each type of quadrilateral—*parallelogram*, *rectangle*, *rhombus*, and *square*—for which the statement is always true.

8. It is equilateral.

9. Its interior angles are all right angles.

10. The diagonals are congruent.

11. Opposite sides are parallel.

12. The vertices of quadrilateral *PQRS* are *P*(−2, 0), *Q*(0, 3), *R*(6, −1), and *S*(1, −2). Draw *PQRS* in a coordinate plane. Show that it is a trapezoid.

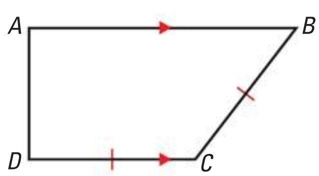
13. One side of a quadrilateral *JKLM* is longer than another side.

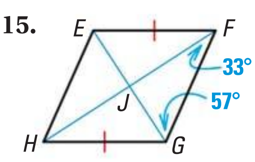
a. Suppose *JKLM* is an isosceles trapezoid. In a coordinate plane, find possible coordinates for the vertices of *JKLM*. *Justify* your answer.

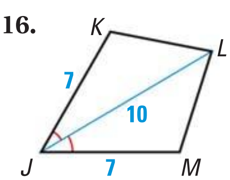
b. Suppose *JKLM* is a kite. In a coordinate plane, find possible coordinates for the vertices of *JKLM*. *Justify* your answer.

c. Name other special quadrilaterals that *JKLM* could be.

Give the most specific name for the quadrilateral. *Explain* your reasoning.

14.


15.


16.


17. In trapezoid *WXYZ*, $\overline{WX} \parallel \overline{YZ}$, and $YZ = 4.25$ centimeters. The midsegment of trapezoid *WXYZ* is 2.75 centimeters long. Find *WX*.

18. In ▱*RSTU*, $\overline{RS}$ is 3 centimeters shorter than $\overline{ST}$. The perimeter of ▱*RSTU* is 42 centimeters. Find *RS* and *ST*.

8 ALGEBRA REVIEW

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GRAPH NONLINEAR FUNCTIONS

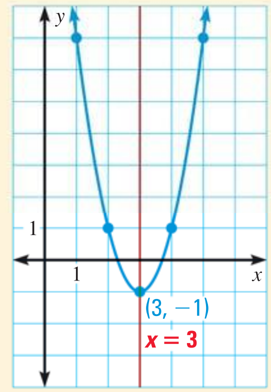
EXAMPLE 1 Graph a quadratic function in vertex form

Graph $y = 2(x - 3)^2 - 1$.

The *vertex form* of a quadratic function is $y = a(x - h)^2 + k$. Its graph is a parabola with vertex at (h, k) and axis of symmetry $x = h$.

The given function is in vertex form. So, $a = 2$, $h = 3$, and $k = -1$. Because $a > 0$, the parabola opens up.

Graph the vertex at $(3, -1)$. Sketch the axis of symmetry, $x = 3$. Use a table of values to find points on each side of the axis of symmetry. Draw a parabola through the points.

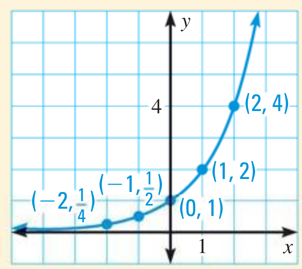


x	3	1	2	4	5
y	−1	7	1	1	7

EXAMPLE 2 Graph an exponential function

Graph $y = 2^x$.

Make a table by choosing a few values for x and finding the values for y. Plot the points and connect them with a smooth curve.



x	−2	−1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

EXERCISES

EXAMPLE 1
for Exs. 1–6

Graph the quadratic function. Label the vertex and sketch the axis of symmetry.

1. $y = 3x^2 + 5$ **2.** $y = -2x^2 + 4$ **3.** $y = 0.5x^2 - 3$

4. $y = 3(x + 3)^2 - 3$ **5.** $y = -2(x - 4)^2 - 1$ **6.** $y = \frac{1}{2}(x - 4)^2 + 3$

EXAMPLE 2
for Exs. 7–10

Graph the exponential function.

7. $y = 3^x$ **8.** $y = 8^x$ **9.** $y = 2.2^x$ **10.** $y = \left(\frac{1}{3}\right)^x$

Use a table of values to graph the cubic or absolute value function.

11. $y = x^3$ **12.** $y = x^3 - 2$ **13.** $y = 3x^3 - 1$

14. $y = 2|x|$ **15.** $y = 2|x| - 4$ **16.** $y = -|x| - 1$

8 ★ Standardized TEST PREPARATION

CONTEXT-BASED MULTIPLE CHOICE QUESTIONS

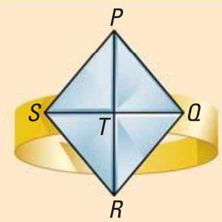
Some of the information you need to solve a context-based multiple choice question may appear in a table, a diagram, or a graph.

PROBLEM 1

Which of the statements about the rhombus-shaped ring is not always true?

Ⓐ $m\angle SPT = m\angle TPQ$ Ⓑ $PT = TR$

Ⓒ $m\angle STR = 90°$ Ⓓ $PR = SQ$



Plan

INTERPRET THE DIAGRAM The diagram shows rhombus *PQRS* with its diagonals intersecting at point *T*. Use properties of rhombuses to figure out which statement is not always true.

Solution

STEP 1 Evaluate choice A.

Consider choice A: $m\angle SPT = m\angle TPQ$.

Each diagonal of a rhombus bisects each of a pair of opposite angles. The diagonal $\overline{PR}$ bisects $\angle SPQ$, so $m\angle SPT = m\angle TPQ$. Choice A is true.

STEP 2 Evaluate choice B.

Consider choice B: $PT = TR$.

The diagonals of a parallelogram bisect each other. A rhombus is also a parallelogram, so the diagonals of *PQRS* bisect each other. So, $PT = TR$. Choice B is true.

STEP 3 Evaluate choice C.

Consider choice C: $m\angle STR = 90°$.

The diagonals of a rhombus are perpendicular. *PQRS* is a rhombus, so its diagonals are perpendicular. Therefore, $m\angle STR = 90°$. Choice C is true.

STEP 3 Evaluate choice D.

Consider choice D: $PR = SQ$.

If the diagonals of a parallelogram are congruent, then it is a rectangle. But *PQRS* is a rhombus. Only in the special case where it is also a square (a type of rhombus that is also a rectangle), would choice D be true. So, choice D is not always true.

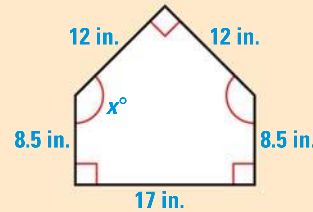
The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

PROBLEM 2

The official dimensions of home plate in professional baseball are shown on the diagram. What is the value of x?

Ⓐ 90 Ⓑ 108

Ⓒ 135 Ⓓ 150



Plan

INTERPRET THE DIAGRAM From the diagram, you can see that home plate is a pentagon. Use what you know about the interior angles of a polygon and the markings given on the diagram to find the value of x.

Solution

STEP 1 Find the sum of the measures of the interior angles.

Home plate has 5 sides. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$(n - 2) \cdot 180° = (5 - 2) \cdot 180°$ **Substitute 5 for *n*.**

$= 3 \cdot 180°$ **Subtract.**

$= 540°$ **Multiply.**

STEP 2 Write and solve an equation.

From the diagram, you know that three interior angles are right angles. The two other angles are congruent, including the one whose measure is $x°$. Use this information to write an equation. Then solve the equation.

$3 \cdot 90° + 2 \cdot x° = 540°$ **Write equation.**

$270 + 2x = 540$ **Multiply.**

$2x = 270$ **Subtract 270 from each side.**

$x = 135$ **Divide each side by 2.**

The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

PRACTICE

In Exercises 1 and 2, use the part of the quilt shown.

1. What is the value of x?

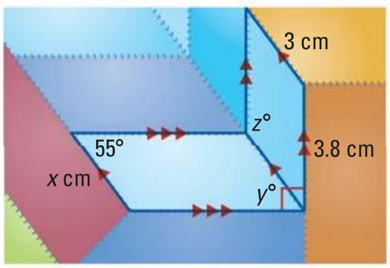
   Ⓐ 3 Ⓑ 3.4

   Ⓒ 3.8 Ⓓ 5.5

2. What is the value of z?

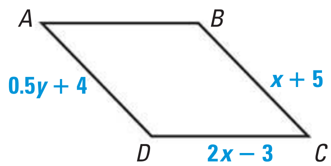
   Ⓐ 35 Ⓑ 55

   Ⓒ 125 Ⓓ 145



MULTIPLE CHOICE

In Exercises 1 and 2, use the diagram of rhombus *ABCD* below.



1. What is the value of x?

Ⓐ 2 Ⓑ 4.6

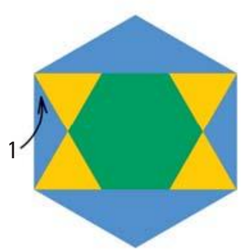
Ⓒ 8 Ⓓ 13

2. What is the value of y?

Ⓐ 1.8 Ⓑ 2

Ⓒ 8 Ⓓ 18

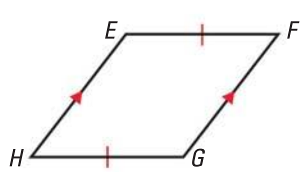
3. In the design shown below, a green regular hexagon is surrounded by yellow equilateral triangles and blue isosceles triangles. What is the measure of $\angle 1$?



Ⓐ 30° Ⓑ 40°

Ⓒ 50° Ⓓ 60°

4. Which statement about *EFGH* can be concluded from the given information?



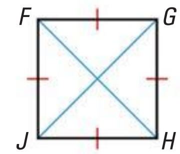
Ⓐ It is not a kite.

Ⓑ It is not an isosceles trapezoid.

Ⓒ It is not a square.

Ⓓ It is not a rhombus.

5. What is the most specific name for quadrilateral *FGHJ*?



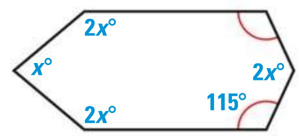
Ⓐ Parallelogram

Ⓑ Rhombus

Ⓒ Rectangle

Ⓓ Square

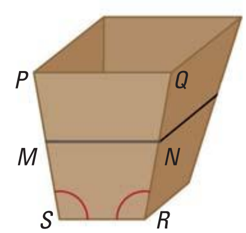
6. What is the measure of the smallest interior angle of the hexagon shown?



Ⓐ 50° Ⓑ 60°

Ⓒ 70° Ⓓ 80°

In Exercises 7 and 8, use the diagram of a cardboard container. In the diagram, $\angle S \cong \angle R$, $\overline{PQ} \parallel \overline{SR}$, and $\overline{PS}$ and $\overline{QR}$ are not parallel.



7. Which statement is true?

Ⓐ $PR = SQ$

Ⓑ $m\angle S + m\angle R = 180°$

Ⓒ $PQ = 2 \cdot SR$

Ⓓ $PQ = QR$

8. The bases of trapezoid *PQRS* are $\overline{PQ}$ and $\overline{SR}$, and the midsegment is $\overline{MN}$. Given $PQ = 9$ centimeters, and $MN = 7.2$ centimeters, what is SR?

Ⓐ 5.4 cm Ⓑ 8.1 cm

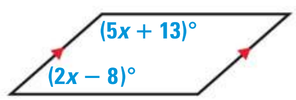
Ⓒ 10.8 cm Ⓓ 12.6 cm

GRIDDED ANSWER

9. How many degrees greater is the measure of an interior angle of a regular octagon than the measure of an interior angle of a regular pentagon?

10. Parallelogram *ABCD* has vertices $A(-3, -1)$, $B(-1, 3)$, $C(4, 3)$, and $D(2, -1)$. What is the sum of the *x*- and *y*-coordinates of the point of intersection of the diagonals of *ABCD*?

11. For what value of *x* is the quadrilateral shown below a parallelogram?

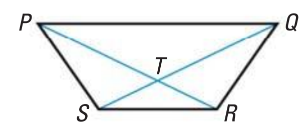


12. In kite *JKLM*, the ratio of *JK* to *KL* is 3 : 2. The perimeter of *JKLM* is 30 inches. Find the length (in inches) of $\overline{JK}$.

SHORT RESPONSE

13. The vertices of quadrilateral *EFGH* are $E(-1, -2)$, $F(-1, 3)$, $G(2, 4)$, and $H(3, 1)$. What type of quadrilateral is *EFGH*? *Explain*.

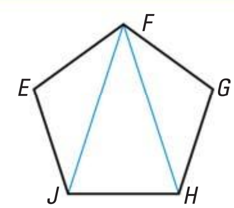
14. In the diagram below, *PQRS* is an isosceles trapezoid with $\overline{PQ} \parallel \overline{RS}$. *Explain* how to show that $\triangle PTS \cong \triangle QTR$.



15. In trapezoid *ABCD*, $\overline{AB} \parallel \overline{CD}$, $\overline{XY}$ is the midsegment of *ABCD*, and $\overline{CD}$ is twice as long as $\overline{AB}$. Find the ratio of *XY* to *AB*. *Justify* your answer.

EXTENDED RESPONSE

16. The diagram shows a regular pentagon and diagonals drawn from vertex *F*.



a. The diagonals divide the pentagon into three triangles. Classify the triangles by their angles and side measures. *Explain* your reasoning.

b. Which triangles are congruent? *Explain* how you know.

c. For each triangle, find the interior angle measures. *Explain* your reasoning.

17. In parts (a)–(c), you are given information about a quadrilateral with vertices *A*, *B*, *C*, *D*. In each case, *ABCD* is a different quadrilateral.

a. Suppose that $\overline{AB} \parallel \overline{CD}$, $AB = DC$, and $\angle C$ is a right angle. Draw quadrilateral *ABCD* and give the most specific name for *ABCD*. *Justify* your answer.

b. Suppose that $\overline{AB} \parallel \overline{CD}$ and *ABCD* has *exactly* two right angles, one of which is $\angle C$. Draw quadrilateral *ABCD* and give the most specific name for *ABCD*. *Justify* your answer.

c. Suppose you are given only that $\overline{AB} \parallel \overline{CD}$. What additional information would you need to know about $\overline{AC}$ and $\overline{BD}$ to conclude that *ABCD* is a rhombus? *Explain*.